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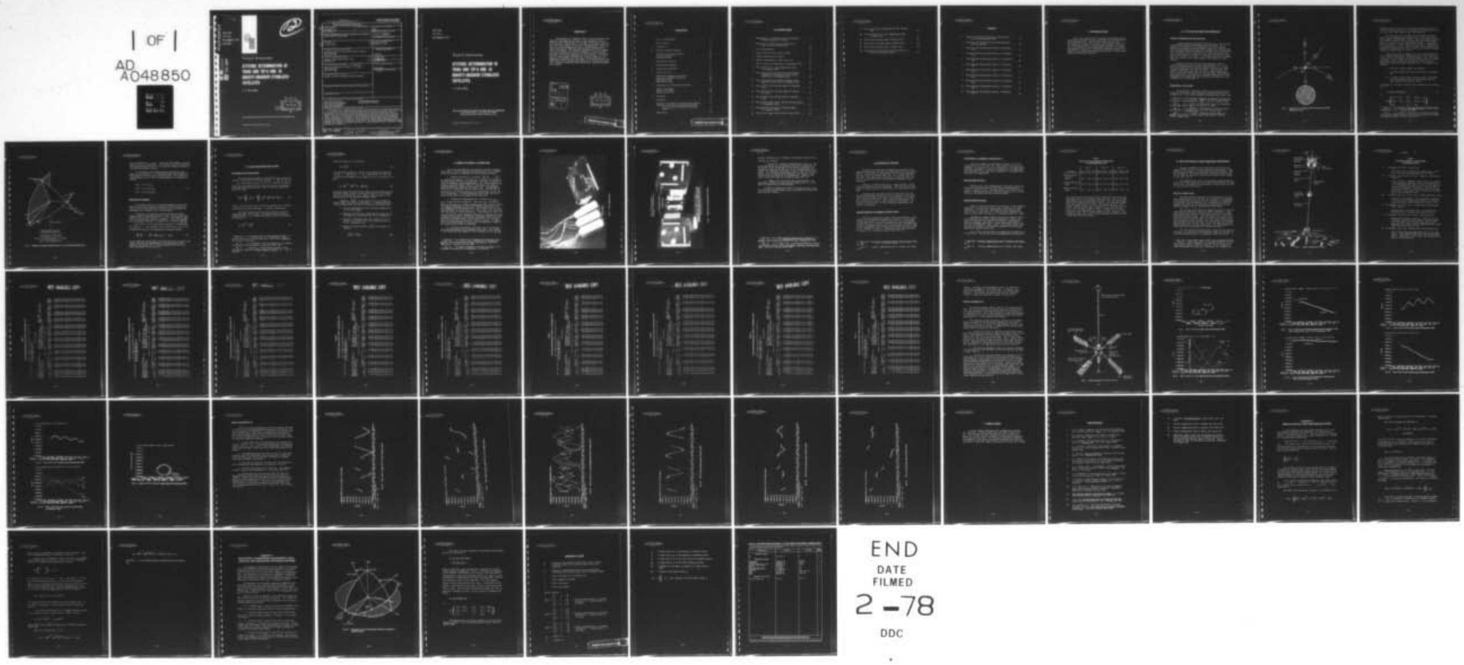

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Technical Memorandum

ATTITUDE DETERMINATION OF TRIAD AND TIP-II AND -III GRAVITY-GRADIENT-STABILIZED SATELLITES

C. E. WILLIAMS

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REPORT DOCUMENTATION PAGE

14

1. REPORT NUMBER: APL/JHU/TG-1313
2. GOVT ACCESSION NO
3. RECIPIENT'S CATALOG NUMBER

9

6

4. TITLE (and Subtitle): ATTITUDE DETERMINATION OF TRIAD AND TIP-II AND -III GRAVITY-GRADIENT-STABILIZED SATELLITES.
5. TYPE OF REPORT & PERIOD COVERED: Technical Memorandum
6. PERFORMING ORG. REPORT NUMBER

10

7. AUTHOR(s): C. E./Williams

15

8. CONTRACT OR GRANT NUMBER(s): N00017-72-C-4401
9. PERFORMING ORGANIZATION NAME & ADDRESS: The Johns Hopkins University Applied Physics Laboratory, Johns Hopkins Rd., Laurel, MD 20810
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Task S1T1
11. CONTROLLING OFFICE NAME & ADDRESS: Strategic Systems Project Office, SP-24, Washington, DC

11

12. REPORT DATE: Dec 1977
13. NUMBER OF PAGES: 63
14. MONITORING AGENCY NAME & ADDRESS: Naval Plant Representative Office, Johns Hopkins Rd., Laurel, MD 20810

12 61p.

15. SECURITY CLASS. (of this report): Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Least-squares estimation
Satellite attitude estimation
Euler angle determination
Eigenvalue determination
Gravity-gradient satellites
Attitude matrix estimation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The attitude of a satellite refers to the rotational orientation of the spacecraft relative to some reference triad of Cartesian axes (these being, for the type of spacecraft treated here, the orbit radius vector, the normal-to-the-orbit plane, and the vector cross product of the two). Mathematically, the attitude is usually represented by nine direction cosines and/or three Euler angles. The numerical determination of these parameters is the objective of attitude estimation. Various schemes have been developed and used by the Applied Physics Laboratory to determine the attitude performance of its satellites. In recent years, a least-squares technique that involves eigenvalue and eigenvector computation has been added. This report presents the formulation of the technique and discusses its successful application. Attitude estimation results from three orbiting spacecraft are included.

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ABSTRACT

The attitude of a satellite refers to the rotational orientation of the spacecraft relative to some reference triad of Cartesian axes (these being, for the type of spacecraft treated here, the orbit radius vector, the normal-to-the-orbit plane, and the vector cross product of the two). Mathematically, the attitude is usually represented by nine direction cosines and/or three Euler angles. The numerical determination of these parameters is the objective of attitude estimation. Various schemes have been developed and used by the Applied Physics Laboratory to determine the attitude performance of its satellites. In recent years, a least-squares technique that involves eigenvalue and eigenvector computation has been added. This report presents the formulation of the technique and discusses its successful application. Attitude estimation results from three orbiting spacecraft are included.

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1. INTRODUCTION

This report presents the formulation and application of a least-squares attitude estimation technique for gravity-gradient-stabilized spacecraft. The following topics are included: (a) definition of satellite attitude, (b) statement of the attitude estimation problem, (c) least-squares solution (see Appendix A for derivation), (d) computational algorithm for solution implementation (including the auxiliary equations derived in Appendix B), (e) sources of estimation error, and (f) results from several orbiting APL satellites.

2. ATTITUDE ESTIMATION PROBLEM

GRAVITY-GRADIENT STABILIZATION

A gravity-gradient-stabilized satellite has one of its axes (usually the Z-axis) always pointed toward the earth. Such a spacecraft is designed to take advantage of the fact that the earth's gravitational field will tend to stabilize a triaxial body (one with unequal principal moments of inertia), with its principal axis of minimum inertia aligned with the local vertical and its axis of maximum inertia aligned with the normal-to-the-orbit plane (see Fig. 1) (Refs. 1, 2, and 3). (The local vertical is an imaginary line from the earth's mass center to the satellite's mass center.)

Spacecraft built by APL have used extendible booms to achieve a favorable moment-of-inertia distribution, i.e., an inertia ellipsoid where the smallest principal inertia is at least an order of magnitude less than the others. The satellites discussed in this report have also included a constant-speed rotor with its spin axis aligned with (or, in some cases, defining) the spacecraft Y-axis. The addition of the wheel enhances the overall stabilization by adding gyroscopic stiffness and stability to the alignment of the Y-axis (Ref. 4).

SPACECRAFT ATTITUDE

The attitude of a satellite refers to the rotational orientation of the satellite axes relative to some reference triad of Cartesian axes. For a gravity-gradient-stabilized spacecraft, this

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Ref. 1. R. E. Fischell, "Magnetic and Gravity Attitude Stabilization of Earth Satellites," ARS J., Vol. 31, September 1961.

Ref. 2. R. A. Nidley, "Gravitational Torque on a Satellite of Arbitrary Shape," ARS J., Vol. 30, No. 2, 1960.

Ref. 3. R. E. Roberson, "Gravitational Torque on a Satellite Vehicle," J. Franklin Inst., Vol. 265, January 1958.

Ref. 4. V. L. Pisacane, "Three-Axis Stabilization of a Dumbbell Satellite by a Small Constant-Speed Rotor," APL/JHU TG 855, October 1966.

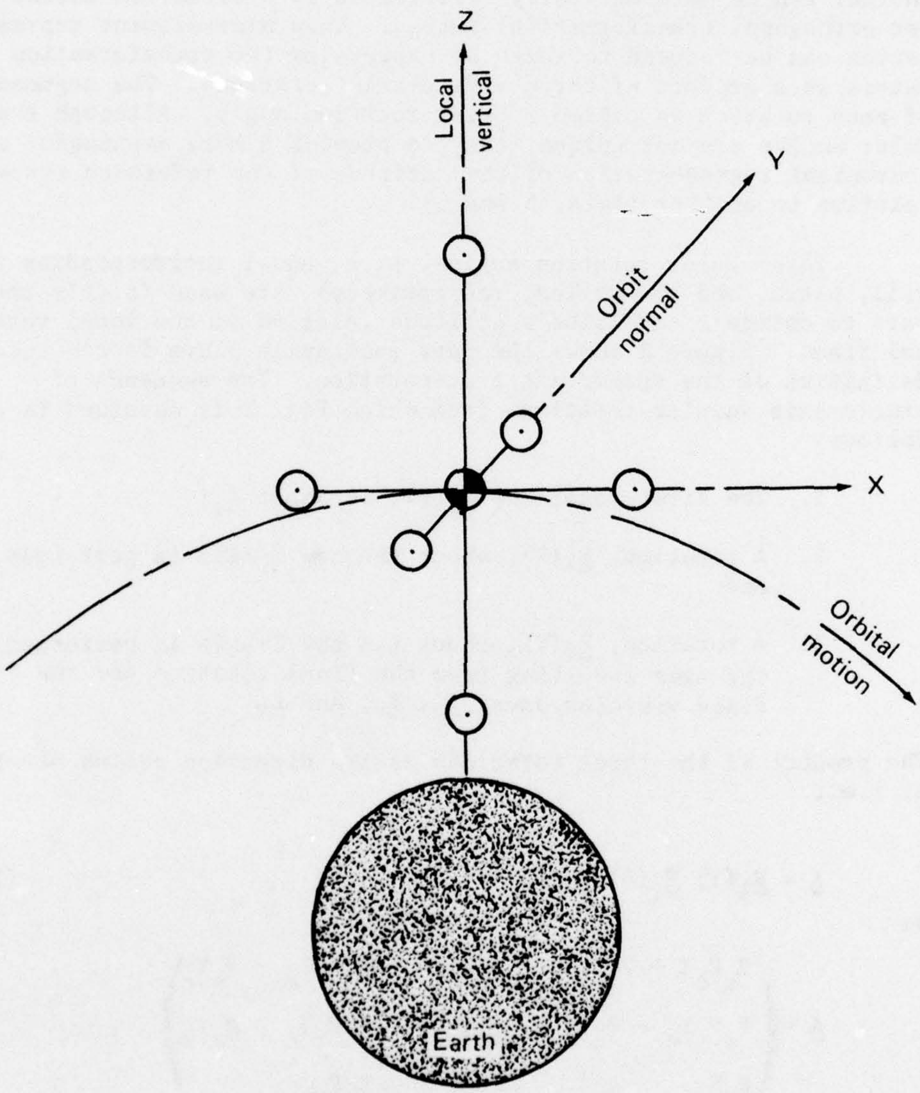


Fig. 1 Stabilization of a Satellite with Three Unequal Principal Moments of Inertia

reference system is called the local vertical system ($\underline{Z}_\ell$ is the outbound local vertical, $\underline{Y}_\ell$ is the normal-to-the-orbit plane, and $\underline{X}_\ell$ is the vector that completes the right-hand set).

The orientation (or attitude) of one reference system to another can be mathematically represented by a direction cosine (or orthogonal transformation) matrix. This nine-element representation can be reduced to three by expressing the transformation matrix as a product of three single-axis rotations. The argument of each rotation is called a Euler rotation angle. Although the Euler angles are not unique, they do provide a more meaningful and convenient representation of the attitude of one reference system relative to another (Refs. 5 and 6).

Three Euler rotation angles, R, P, and Y (corresponding to roll, pitch, and yaw angles, respectively), are used in this report to define a satellite's attitude relative to the local vertical frame. Figure 2 shows the part each angle plays in the total definition of the spacecraft's orientation. The sequence of single-axis angular rotations from which Fig. 2 is obtained is as follows:

1. The first rotation, $\underline{\underline{R}}_2(P)$, is about $\underline{Y}_\ell$;

2. A rotation, $\underline{\underline{R}}_1(R)$, about the new X-axis is performed, and

3. A rotation, $\underline{\underline{R}}_3(Y)$, about the new Z-axis is performed. The axes resulting from the final rotation are the fixed vehicles axes, $\underline{X}_v$, $\underline{Y}_v$, and $\underline{Z}_v$.

The product of the three rotations is the direction cosine matrix, $\underline{\underline{A}}$, i.e.,

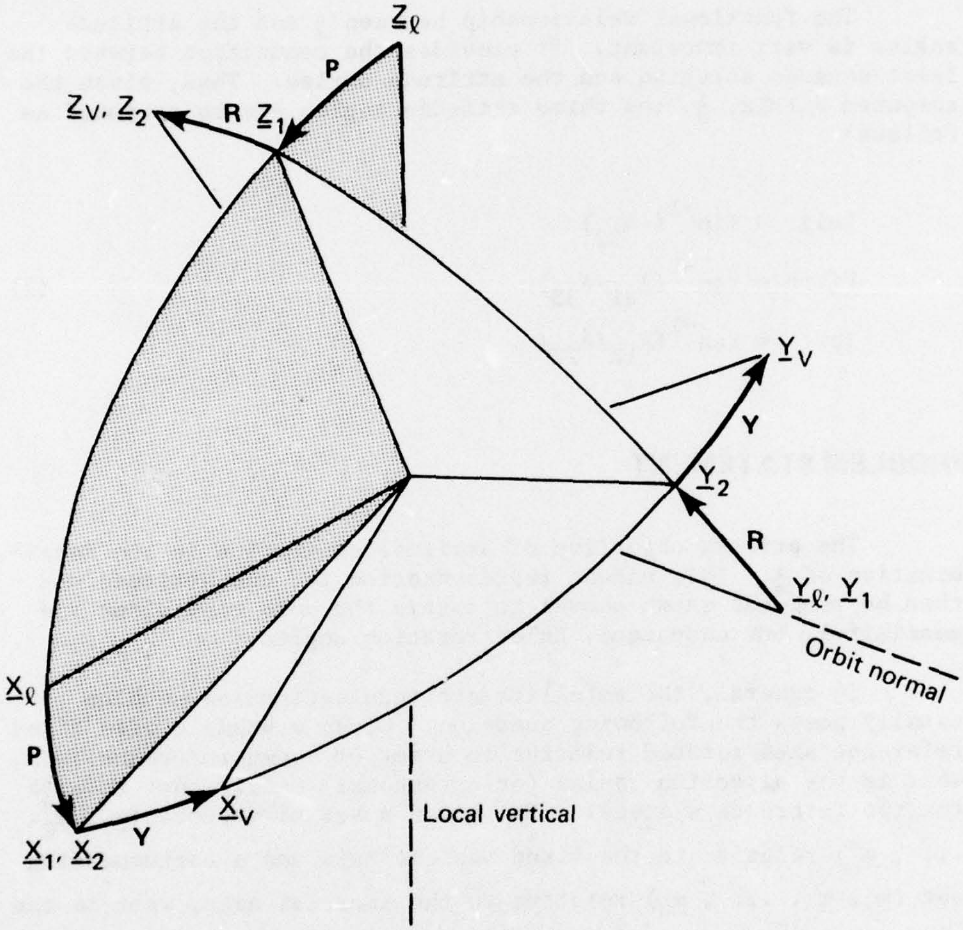
$$\underline{\underline{A}} = \underline{\underline{R}}_3(Y)\ \underline{\underline{R}}_1(R)\ \underline{\underline{R}}_2(P) \qquad (1)$$

or

$$\underline{\underline{A}} = \begin{pmatrix} R_s P_s Y + P_c Y_c & R_c Y_s & R_s P_c Y_s - P_s Y_c \\ R_s P_s Y_c - P_c Y_s & R_c Y_c & R_s P_c Y_c + P_s Y_s \\ R_c P_s & -R_s & R_c P_c \end{pmatrix} ,$$

Ref. 5. H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Co., Inc., Reading, MA, 1950.

Ref. 6. G. A. Smith, "Four Methods of Attitude Determination for Spin-Stabilized Spacecraft with Applications and Comparative Results," NASA, TR R-445, August 1975.



Transformation from local vertical axes to vehicle axes

$$U_{veh} = \begin{bmatrix} Y_c & Y_s & 0 \\ -Y_s & Y_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & R_c & R_s \\ 0 & -R_s & R_c \end{bmatrix} \begin{bmatrix} P_c & 0 & -P_s \\ 0 & 1 & 0 \\ P_s & 0 & P_c \end{bmatrix} U_{local}$$

Fig. 2 Definition of Satellite Attitude Relative to Local Vertical Reference Axes

where the subscripts s and c denote the trigonometric sine and cosine functions, respectively. By definition, $\underline{\underline{A}}$ is the orthogonal transformation matrix from the local vertical reference system to the satellite reference system.

The functional relationship between $\underline{\underline{A}}$ and the attitude angles is very important. It provides the connection between the least-squares solution and the attitude angles. Thus, given the computed matrix, $\underline{\underline{A}}$, the three attitude angles can be computed as follows:

$$\begin{aligned} \text{Roll} &= \sin^{-1}(-A_{32}) \\ \text{Pitch} &= \tan^{-1}(A_{31}/A_{33}) \\ \text{Yaw} &= \tan^{-1}(A_{12}/A_{22}) \ . \end{aligned} \tag{2}$$

PROBLEM STATEMENT

The primary objective of attitude estimation is the determination of $\underline{\underline{A}}$. This unique representation for the attitude can then be used (as shown above) to obtain the more convenient and meaningful, but nonunique, Euler rotation angles.

In general, the satellite attitude estimation problem usually poses the following question: Given a vehicle with fixed reference axes rotated relative to a set of known reference axes, what is the direction cosine (or orthogonal) matrix that relates the two reference systems? Or, given a set of vectors $(\underline{m}_1^*, \underline{m}_2^*, \ldots, \underline{m}_n^*)$ relative to the fixed vehicle axis and a corresponding set $(\underline{m}_1, \underline{m}_2, \ldots, \underline{m}_n)$ relative to the inertial axes, what is the orthogonal matrix, $\underline{\underline{A}}$, that satisfies the following equation

$$(\underline{m}_1^*, \underline{m}_2^*, \ldots, \underline{m}_n^*) = \underline{\underline{A}}(\underline{m}_1, \underline{m}_2, \ldots, \underline{m}_n) \ ? \tag{3}$$

In most satellite applications, the first vector set is usually the output resulting from measurements by on-board sensors (star trackers, sun sensors, etc.), and the second vector set is the known counterpart of the first set.

3. LEAST-SQUARES SOLUTION

MATHEMATICAL EQUATIONS

The least-squares approach to the problem can be stated as follows: Given the set of vectors $(\underline{m}_1^*, \underline{m}_2^*, \ldots, \underline{m}_n^*)$ and $(\underline{m}_1, \underline{m}_2, \ldots, \underline{m}_n)$ as defined previously (for $n \geq 2$), find the orthogonal matrix, $\underline{\underline{A}}$, that brings the first set into the best least-squares coincidence with the second set. That is, find an $\underline{\underline{A}}$ that minimizes the scalar

$$Q(\underline{\underline{A}}) = \sum_{j=1}^{n} \underline{e}_j^T \underline{e}_j = \sum_{j=1}^{n} (\underline{m}_j^* = \underline{\underline{A}}\underline{m}_j)^T (\underline{m}_j^* - \underline{\underline{A}}\underline{m}_j) \quad , \qquad (4)$$

where $\underline{e}_j$ is the column vector of errors associated with the $\underline{m}_n^*$ observed vector and the superscript T denotes transposition.

This least-squares problem has been treated in the open literature (Refs. 7, 8, and 9). In Appendix A the derivation (taken from Ref. 7) of a closed form solution for $\underline{\underline{A}}$ is discussed. The solution is

$$\underline{\underline{A}} = (\underline{\underline{P}}^T)^{-1} (\underline{\underline{P}}^T \underline{\underline{P}})^{1/2} \qquad (5)$$

Ref. 7. J. L. Farrell and J. C. Stuelpnagel, "A Least Squares Estimate of Satellite Attitude," Problem 6501, SIAM Rev., Vol. 8, No. 3, July 1965.

Ref. 8. P. B. Davenport, "A Vector Approach to the Algebra of Rotations with Applications," NASA, TN D-4696, 1968.

Ref. 9. L. Fraiture, "A Least-Squares Estimate of the Attitude of a Satellite," AIAA J. Spacecr. Rockets, Vol. 7, No. 5, May 1970.

where the matrix, $\underline{\underline{P}}$, is defined as

$$\underline{\underline{P}} = \underline{\underline{M}}^* \underline{\underline{M}}^T \tag{6}$$

and the matrices, $\underline{\underline{M}}^*$ and $\underline{\underline{M}}$, consist of the juxtaposed column vectors $\underline{m}_1^*, \ldots, \underline{m}_n^*$ and $\underline{m}_1, \ldots, \underline{m}_n$, respectively. For the case in which the determinant of $\underline{\underline{P}}$ is less than zero, the solution for $\underline{\underline{A}}$ is

$$\underline{\underline{A}} = (\underline{\underline{P}}^T)^{-1} (\underline{\underline{P}}^T\underline{\underline{P}})^{\frac{1}{2}} (\underline{\underline{I}} - 2\underline{\underline{G}}^T\underline{\underline{H}}\ \underline{\underline{G}}) , \tag{7}$$

where $\underline{\underline{G}}$ is the model matrix whose columns are the eigenvectors of the matrix $(\underline{\underline{P}}^T\underline{\underline{P}})$, and $\underline{\underline{H}}$ is an nth-order matrix with all elements equal to zero except the (n, n) element, which is equal to one. The matrix, $\underline{\underline{I}}$, is the nth-order identity matrix.

The matrix, $(\underline{\underline{P}}^T\underline{\underline{P}})^{\frac{1}{2}}$, is the square root of the symmetric matrix, $(\underline{\underline{P}}^T\underline{\underline{P}})$, with positive eigenvalues. A computational algorithm for determining this matrix contains the following steps:

1. Find the eigenvalues and the normalized eigenvectors of the matrix $(\underline{\underline{P}}^T\underline{\underline{P}})$;

2. Construct a matrix, $\underline{\underline{G}}$, in which the kth column is the kth eigenvector associated with the kth eigenvalue;

3. Construct a diagonal matrix, $\underline{\underline{D}}$, in which the (k, k) element is the square root of the absolute value of the kth eigenvalue; and

4. Compute the desired matrix, $(\underline{\underline{P}}^T\underline{\underline{P}})^{\frac{1}{2}}$, according to the equation

$$(\underline{\underline{P}}^T\underline{\underline{P}})^{\frac{1}{2}} = \underline{\underline{G}}^T\underline{\underline{D}}\ \underline{\underline{G}} . \tag{8}$$

4. COMPUTATIONAL ALGORITHM

The actual determination of $\underline{\underline{A}}$ involves a series of computational steps. For the applications discussed here, the following steps were required to implement the least-squares technique:

1. Compute the first set of vectors, $(\underline{m}_1^*, \underline{m}_2^*, \ldots, \underline{m}_n^*)$, in satellite coordinates. The first set of vectors is provided by attitude sensors on board the spacecraft. Two types of sensors, a triad of orthogonal vector magnetometers (Fig. 3) and digital solar attitude detectors (DSAD's) (Fig. 4), were used on the gravity-gradient-stabilized satellites discussed later. The geomagnetic field vector (denoted $\underline{m}_1^*$) and the sunline vector (denoted $\underline{m}_2^*$) are obtained from the output signals of the magnetometers and sun sensors, respectively. A third independent vector, $\underline{m}_3^*$, can be computed as the vector cross product of the other two.

2. Compute the corresponding set of vectors, $(\underline{m}_1, \underline{m}_2, \ldots, \underline{m}_n)$ in local vertical coordinates. The vector, $\underline{m}_1$, is obtained from the evaluation of a complex mathematical model (48-term spherical harmonic expansion) of the earth's magnetic field (Ref. 10). The computation of the sunline vector, $\underline{m}_2$, is based upon the cataloged ephemeris of the sun (Ref. 11). As in step 1, the cross product of $\underline{m}_1$ and $\underline{m}_2$ is used to provide $\underline{m}_3$. The satellite's orbit is a required input in these computations because the mathematical formulations are referenced to an inertial coordinate system called the geocentric reference system (Z is the North Pole, X is the first point of Aries, and Y is the vector cross of Z and X). In Appendix B, the transformation of vectors from the geocentric system to the local vertical system is discussed.

3. Construct the matrices, $\underline{\underline{M}}^*$ and $\underline{\underline{M}}$. Two 3 by 3 matrices are constructed using the two computed vector sets. The columns of the matrix, $\underline{\underline{M}}^*$, are composed of the juxtaposed vectors, $\underline{m}_1^*$, $\underline{m}_2^*$,

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Ref. 10. J. C. Cain et al., "Computation of the Main Geomagnetic Field from Spherical Harmonic Expansions," NASA/GSFC, NSSDC 68-11, Greenbelt, MD, May 1968.

Ref. 11. The American Ephemeris and Nautical Almanac, U.S. Government Printing Office, Washington, DC, 1972-1977.

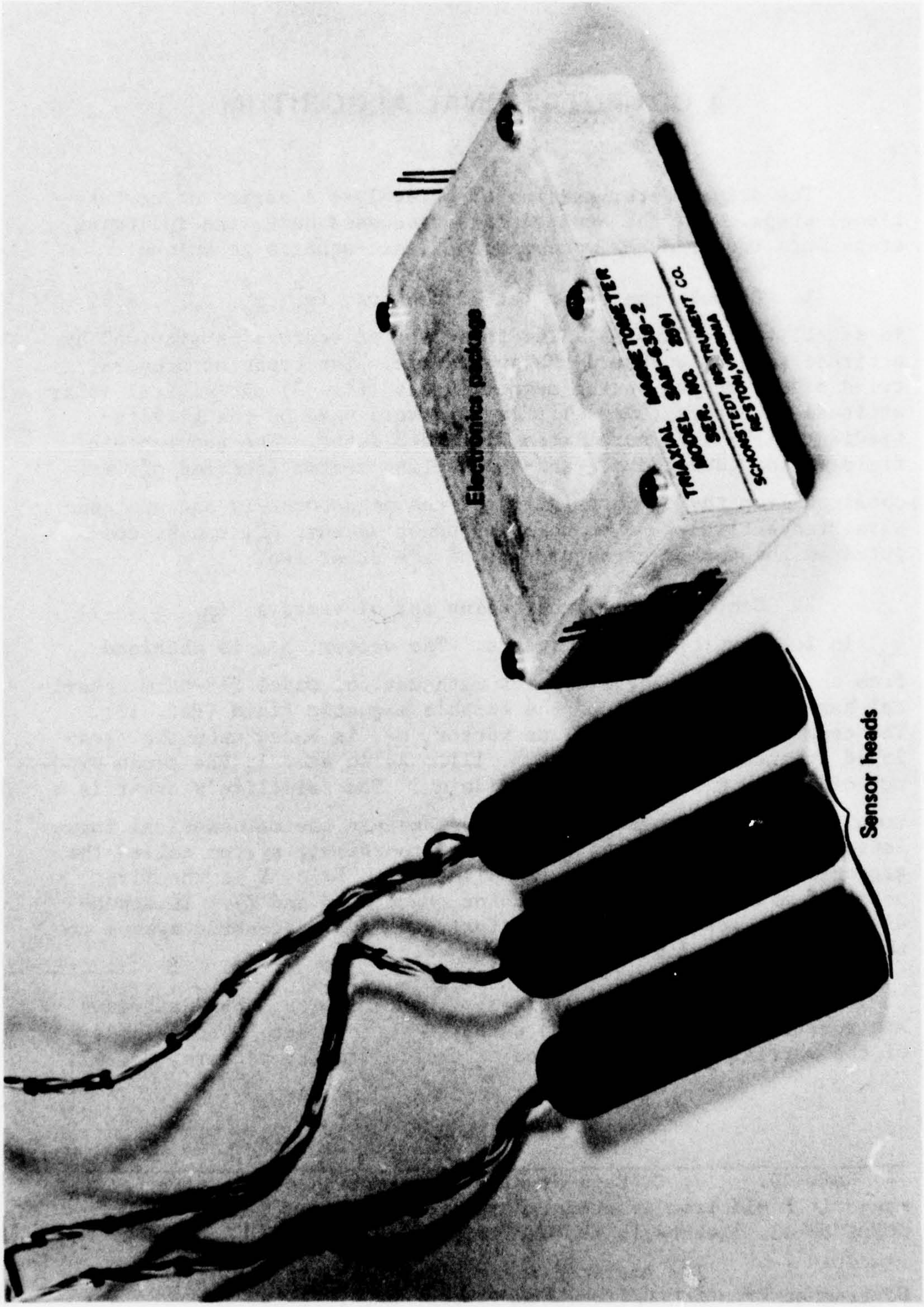

Fig. 3 Vector Magnetometers

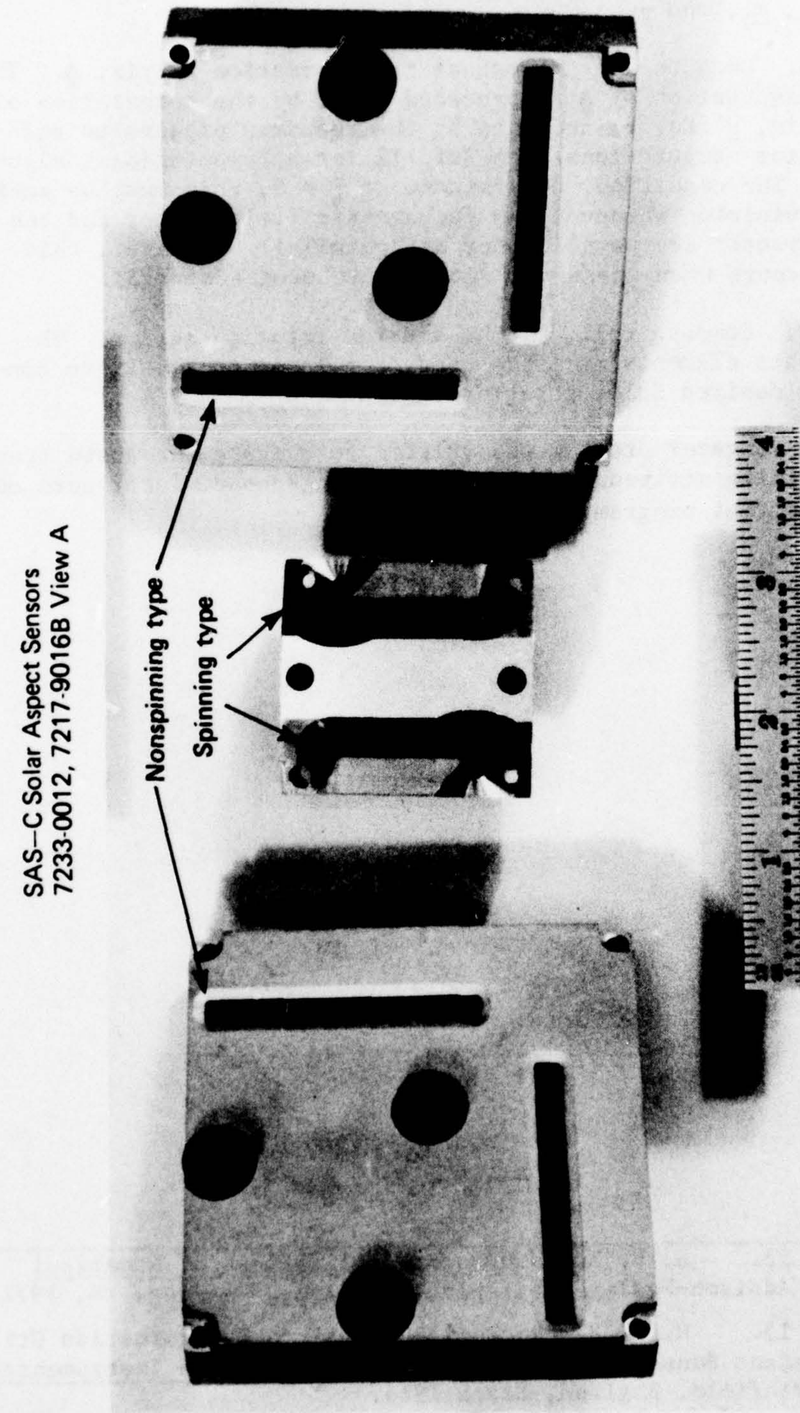

Fig. 4 Digital Solar Attitude Detector (DSAD)

and $\underline{m}_3^*$. The matrix, $\underline{\underline{M}}$, is similarly constructed, using the vectors, $\underline{m}_1$, $\underline{m}_2$, and $\underline{m}_3$.

4. Compute the orthogonal transformation matrix, $\underline{\underline{A}}$. The actual computation of $\underline{\underline{A}}$ is preceded first by the computation of the matrix, $\underline{\underline{P}}$ (Eq. 6) and then by the required eigenvalue and eigenvector computations (see Ref. 12 for a computational algorithm). The condition, determinant of $\underline{\underline{P}} \neq 0$, that must be satisfied is violated whenever the geomagnetic field vector and the sunline vector are parallel (or antiparallel). However, this rarely occurs when these two vectors are used (Ref. 13).

5. Compute roll, pitch, and yaw rotation angles. The appropriate elements of $\underline{\underline{A}}$ are used, according to Eq. 2, to compute the desired Euler rotation angles.

A computer program was written to execute the above steps. The satellite attitude results that are discussed later were outputs from that program.

Ref. 12. S. S. Kuo, Computer Applications of Numerical Methods, Addison-Wesley Publishing Co., Inc., Reading, MA, 1972.

Ref. 13. H. D. Black et al., "Attitude Determination Utilizing Redundant Sensors," Proc. 4th Intern. Aerospace Instrumentation Symp., Cranfield, England, March 1966.

5. SOURCES OF ERROR

The types of errors that can degrade the accuracy of the least-squares estimate include errors associated with the attitude sensors. Inherent in the least-squares theory is the assumption that the errors are Gaussian (Ref. 14), i.e., random noise with zero mean. This means that the technique is generally most accurate for errors that can be characterized as Gaussian in their statistics.

However, systematic errors (e.g., sensor biases), seldom qualify as Gaussian in their statistics. Thus this type of error poses the greater threat to the accuracy of a least-squares estimate.

Although there are many sources of such errors that can theoretically degrade the accuracy of the estimates of the satellite's attitude, some of these errors are expected to be significant, while others are not. Thus, a thorough discussion of all the errors will not serve a useful purpose in this report. Instead, a brief discussion of the anticipated significant error sources is presented.

MAGNETOMETER ALIGNMENT UNCERTAINTY

There are always small errors, called uncertainties, accompanying the positioning or aligning of a sensor. An uncertainty in this application is the angle between the actual and assumed position vectors of a sensor. Theoretically, the three magnetometers will be aligned with the X, Y, and Z satellite reference axes so that the magnetic field components along these axes can be measured. However, in practice, each sensor can be positioned only to within 0.6° of the desired vehicle axis (Ref. 15).

Ref. 14. R. Deutsch, Estimation Theory, Prentice-Hall, Inc., NJ, 1965.

Ref. 15. Private communication with B. Tossman, APL, March 1971.

SUN SENSOR ALIGNMENT UNCERTAINTY

Each of the DSAD's can be mounted to within 0.1° of its assumed position vector (Ref. 16). The results of the error are analogous, in description, to those attributed to the magnetometer alignment uncertainty. However, the magnitude of the DSAD alignment uncertainty is so small that its effect on the attitude estimates will be insignificant.

MAGNETOMETER BIAS

Biases on the vector magnetometers could result from using incorrect calibration tables or from actual residual magnetic dipoles on the spacecraft. Although prelaunch procedures are designed to minimize the problem, postlaunch schemes such as recalibration or bias estimation are usually available.

MAGNETOMETER NOISE

Because of sensor electronics nonlinearity or other inherent limitations, an uncertainty called noise accompanies any sensor measurement. For the vector magnetometers, the expected noise level is about 1 mA/m (Ref. 17). This is usually described as Gaussian noise, with zero mean and a standard deviation of 1 mA/m.

The effects of these errors on attitude estimation results can be examined via digital computer simulations. Table 1 summarizes the results of a study of the effects of magnetometer noise and alignment uncertainty. Three simulations, Cases I, II, and III, were performed. Each covered a time period greater than one orbit (≈ 100 min). Least-squares estimates of the attitude angles were computed every 2 min.

Case I dealt with the effects of magnetometer alignment uncertainty only. The results of the investigation indicated that a

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Ref. 16. Private communication with G. Fountain, APL, March 1971.

Ref. 17. Private communication with F. Mobley, APL, March 1971.

Table 1

Results of Testing Least-Squares Technique with Corrupted Magnetometer Data

Statistics	Case I			Case II			Case III		
	Roll	Pitch	Yaw	Roll	Pitch	Yaw	Roll	Pitch	Yaw
Average Absolute Deviation (deg)	0.07	0.18	0.07	0.21	0.30	0.15	0.41	0.78	0.39
Standard Deviation (deg)	0.04	0.04	0.03	0.14	0.19	0.13	0.33	0.61	0.37
Maximum Deviation (deg)	0.15	0.25	0.12	0.63	0.96	0.58	1.81	3.38	2.13

worst-case position uncertainty (0.6°) would cause less than a one-half degree error in any attitude angle estimate. Case II included magnetometer position uncertainty and noise. The uncertainty numbers were the same as those used in Case I. The noise was Gaussian with a one-sigma rating of 1 mA/m. The maximum increase in the average estimate errors, due to the addition of the noise, was only 0.14°. Case III was the same as Case II, except that the input noise had a one-sigma rating of 5 mA/m. Of course this case produced the largest attitude angle estimate errors. However, on the average the roll, pitch, and yaw angle estimates remained less than 1° away from their true values.

6. APPLICATION OF LEAST-SQUARES TECHNIQUE

Attitude estimation results from several APL spacecraft (Triad, TIP-II, and TIP-III) are presented in this section. Each satellite was designed to be three-axis attitude-stabilized. Each was configured so that gravitational and gyroscopic forces would tend to align the satellite's axes with the local vertical system of axes (see Fig. 1).

The estimated roll, pitch, and yaw angles express quantitatively the angular deviations of the satellite's axes from the local vertical set. The smaller the magnitudes of these angles, the better the attitude stabilization.

RESULTS FROM TRIAD

The Triad satellite (see Fig. 5) was launched into a polar orbit during the fall of 1972 (Ref. 18). Table 2 is a chronology of the significant events prior to and including its achievement of three-axis attitude stabilization. Included in the table are comments that point out the anticipated characteristics of the attitude dynamics that are compatible with each event.

The attitude angles that were computed according to the algorithm outlined earlier are listed in Tables 3 through 11. Each table contains the results from one day's collection of attitude data during passes over APL. The horizontal lines in each table separate the various passes. When the results are grouped according to the events listed in Table 2, it can be seen that the attitude performance was as expected. The best attitude stabilization was observed on the last two days, 271 and 272, when the attitude dynamics appeared to be near steady-state. The angle between the Z-axis and the local vertical was consistently below 5°.

For each attitude estimation result, there are two error indicators. One indicator is the difference between the theoretical and observed angles between the geomegnetic field and sunline

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Ref. 18. Space Dept. Staff of APL and the Guidance and Control Staff of Stanford University, "A Satellite Freed of All but Gravitational Forces: TRIAD I," Paper No. 74-215, presented at AIAA 12th Aerospace Sciences Mtg., Washington, DC, January 1974.

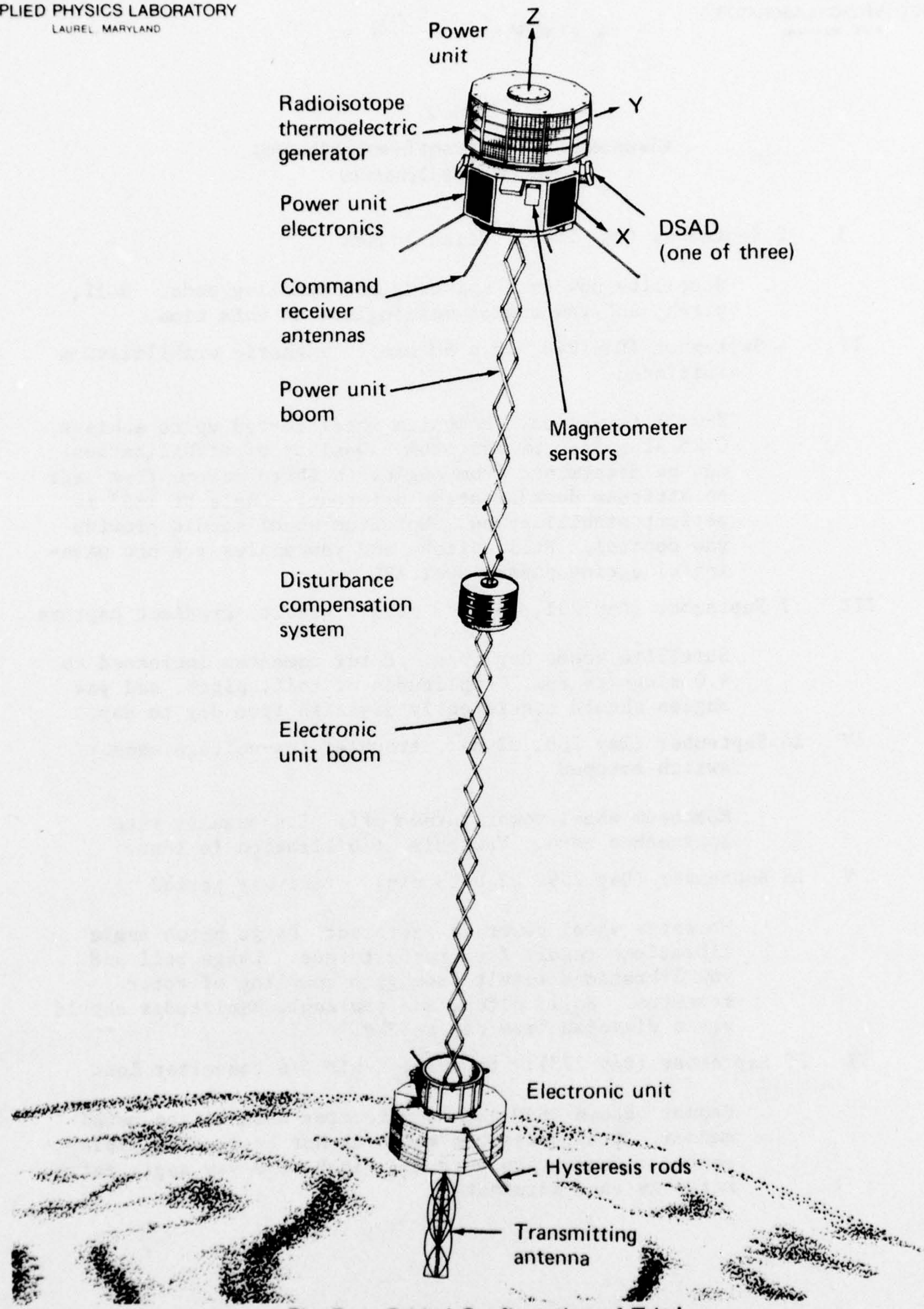

Fig. 5 Orbital Configuration of Triad

Table 2

Chronology of Significant Events Affecting Triad Attitude Dynamics

I 2 September (Day 246): Triad launch

Satellite now in a spinning and tumbling mode. Roll, pitch, and yaw angles meaningless at this time.

II 4 September (Day 248, 02 h 50 min): magnetic stabilization initiated

Z-coil turned on. Momentum wheel revved up to achieve 0.25 slug-ft^2 rpm momentum. Quality of stabilization can be determined from angles in third column from left on attitude determination printout. Angle of 180° is perfect stabilization. Momentum wheel should provide yaw control. Roll, pitch, and yaw angles are now meaningful during passes over APL.

III 7 September (Day 251, 15 h 13 min): gravity-gradient capture

Satellite booms deployed. Rotor momentum increased to 4.0 slug-ft^2 rpm. Amplitudes of roll, pitch, and yaw angles should consistently diminish from day to day.

IV 14 September (Day 258, 22 h): trouble; low-voltage sensor switch tripped

Momentum wheel power turned off. Its angular rate approaches zero. Yaw axis stabilization is lost.

V 15 September (Day 259, 02 h 23 min): recovery period

Momentum wheel power is restored. Large pitch angle librations result from rotor torque. Large roll and yaw librations result from gyro coupling of rotor momentum. Roll, pitch, and yaw angle amplitudes should again diminish from day to day.

VI 29 September (Day 273): trouble; 8-bit A/D converter lost

Cannot obtain DSAD and magnetometer data in the usual manner. Other possible means cannot be readily implemented. Generation of roll, pitch, and yaw angle estimates is thus terminated.

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Table 3

Triad Attitude Determination Results, 8 September 1972

***** COMMENCE ATTITUDE DETERMINATION PROCESSING *****

EULER ANGLE SEQUENCE : POSITIVE PITCH MEANS NOSE DOWN
POSITIVE ROLL MEANS RIGHT WING DOWN
POSITIVE YAW MEANS NOSE LEFT

ORBIT PARAMETERS

EPOCH -TIME = 82.833 (KSECS-UT)	DAY OF EPOCH = 251	YEAR OF EPOCH = 1972
SEMI MAJOR AXIS= 1.123677 (EARTH RADII)	ARG. OF PERIGEE= 333.4968 (DEG)	PERIGEE ALT.= 401.31 (NAUT MI)
ECCENTRICITY= 0.00629	PRECESSION PER.= -3.30108 (DEG/DAY)	APOGEE ALT. = 449.98 (NAUT MI)
INCLINATION= 90.12863 (DEG)	RT. ASCENSION OF NODE AT EPOCH= 310.2959 (DEG)	
ORBIT PERIOD= 100.64 (MIN)	PRECESSION OF NODE= 0.01336 (DEG/DAY)	

STARTING ON DAY 252 AT UT TIME 9811.0 SECS

TIME HRS-MIN-SECS	ESTIMATED ATTITUDE ANGLES IN DEGREES PITCH	ROLL	YAW	ANG BET SUN AND GEOMAG VCTRS (DEG) THEORET	OBSERVD	GEOMAG VCTR MAGNITUDE (MOE) THEORET	OBSERVD	ANG BET VEH Z-AXIS AND FOLLOWING VECTORS IN DEGREES SUNLINE	GEOMAGN	LOCVERT	SATELLT LATITDE DEGREES
2 43 31.0	17.92	-1.57	12.08	54.54	56.11	411.66	411.13	99.24	150.43	17.98	51.30
2 44 54.0	14.86	1.27	28.64	60.21	60.52	415.97	417.85	100.01	155.95	14.92	56.25
4 24 0.0	-4.02	5.15	-25.69	56.86	58.57	413.01	411.54	121.15	167.38	6.53	50.51
13 7 3.0	15.16	-0.67	8.63	130.73	131.04	390.44	388.84	49.45	172.64	15.17	59.97
13 8 26.0	11.86	0.74	2.27	136.13	136.18	384.98	384.55	49.66	172.88	11.88	55.08
13 9 48.0	8.69	0.26	-17.97	141.21	141.16	377.71	374.74	48.30	170.49	8.70	50.25
13 11 11.0	6.07	-1.30	-14.26	145.82	146.32	368.17	368.04	45.42	166.32	6.21	45.36
13 12 33.0	3.92	-1.06	5.28	149.44	151.13	356.50	353.53	43.50	160.72	4.06	40.54
13 13 56.0	0.56	1.26	0.42	151.56	151.75	342.47	336.57	45.31	153.65	1.39	35.65
13 15 18.0	-2.89	1.18	-17.35	151.52	152.80	326.62	323.50	44.21	146.45	3.12	30.83
13 16 41.0	-5.46	0.81	-18.16	149.04	154.04	308.92	304.12	42.21	140.19	5.52	25.94
13 18 4.0	-4.19	4.30	-31.30	144.30	157.66	290.03	258.56	42.06	137.14	6.00	21.06
14 46 9.0	1.55	6.23	12.14	120.43	119.64	411.18	410.61	68.31	171.09	6.42	65.64
14 47 32.0	4.12	6.42	-3.99	125.52	125.88	410.70	411.13	62.58	170.64	7.63	60.75
14 48 54.0	7.15	5.34	-3.67	130.44	130.00	408.29	409.29	56.96	172.36	8.92	55.92
14 50 17.0	10.88	5.13	15.08	135.14	134.77	403.33	400.65	51.57	172.96	12.01	51.03
14 51 40.0	13.64	5.54	8.25	139.31	139.28	395.44	392.88	47.81	172.60	14.70	46.14
14 53 2.0	16.59	4.12	-6.49	142.55	142.76	384.59	383.66	43.21	174.04	17.08	41.31
14 54 25.0	18.75	3.08	-4.94	144.47	144.24	370.54	367.14	40.38	174.65	18.99	36.43
14 55 47.0	20.30	2.16	13.61	144.57	144.64	353.88	347.95	38.29	173.22	20.41	31.61
14 57 10.0	20.55	3.26	15.58	142.53	142.57	334.66	327.74	39.23	168.91	20.80	26.72
14 58 32.0	19.07	2.35	-2.75	138.39	139.46	313.99	307.18	38.08	162.89	19.21	21.90
14 59 55.0	18.26	1.13	-0.65	132.28	132.93	292.20	285.50	37.67	156.37	18.29	17.01
16 28 1.0	-16.39	-2.14	8.82	121.93	122.38	414.65	409.85	72.97	155.71	16.53	61.53
16 29 23.0	-15.97	-1.71	-6.11	126.54	127.57	411.31	408.74	68.80	153.42	16.06	56.70
16 30 46.0	-15.54	-2.13	-12.53	130.89	129.84	404.96	403.41	65.32	151.21	15.68	51.81
16 32 8.0	-14.07	-2.51	1.97	134.64	134.10	395.45	388.89	60.02	149.10	14.29	46.98
16 33 31.0	-12.59	-1.03	1.63	137.50	137.37	382.50	378.76	56.15	147.00	12.63	42.09
16 34 54.0	-10.92	-0.57	-13.75	139.02	138.84	366.42	360.54	51.92	144.51	10.94	37.21
16 36 16.0	-8.47	-0.83	-6.77	138.78	138.30	347.93	340.96	47.10	142.25	8.51	32.38

THE JOHNS HOPKINS UNIVERSITY
APPLIED PHYSICS LABORATORY
LAUREL, MARYLAND

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Table 4

Triad Attitude Determination Results, 10 September 1972

***** COMMENCE ATTITUDE DETERMINATION PROCESSING *****

EULER ANGLE SEQUENCE : POSITIVE PITCH MEANS NOSE DOWN
POSITIVE ROLL MEANS RIGHT WING DOWN
POSITIVE YAW MEANS NOSE LEFT

ORBIT PARAMETERS

EPOCH -TIME = 82.833 (KSECS-UT)
SEMI MAJOR AXIS= 1.123677 (EARTH RADII)
ECCENTRICITY= 0.00629
INCLINATION= 90.12863 (DEG)
ORBIT PERIOD= 100.64 (MIN)

DAY OF EPOCH = 251
ARG. OF PERIGEE= 333.4968 (DEG)
PRECESSION PER.= -3.30108 (DEG/DAY)
RT. ASCENSION OF NODE AT EPOCH= 310.2959 (DEG)
PRECESSION OF NODE= 0.01336 (DEG/DAY)

YEAR OF EPOCH = 1972
PERIGEE ALT.= 401.31 (NAUT MI)
APOGEE ALT. = 449.98 (NAUT MI)

STARTING ON DAY 254 AT UT TIME 6149.0 SECS

TIME HRS-MIN-SECS			ESTIMATED ATTITUDE ANGLES IN DEGREES PITCH	ROLL	YAW	ANG BET SUN AND GEOMAG VCTRS (DEG) THEORET	OBSERVD	GEOMAG VCTR MAGNITUDE (MOE) THEORET	OBSERVD	ANG BET VEH Z-AXIS AND FOLLOWING VECTORS IN DEGREES SUNLINE	GEOMAGN	LOCVERT	SATELLT LATITDE DEGREES
1	42	29.0	6.93	2.53	2.48	49.33	51.27	395.27	391.30	112.37	159.09	7.38	48.77
1	43	52.0	8.09	2.34	3.68	54.98	55.92	402.10	400.67	108.05	160.47	8.42	53.72
1	45	14.0	9.36	1.59	3.76	60.61	61.83	406.28	401.73	102.65	161.78	9.50	58.61
3	24	20.0	-12.77	1.12	-5.29	58.44	60.09	417.11	418.22	122.89	177.02	12.82	52.88
3	25	43.0	-11.92	1.59	-4.49	63.74	65.52	420.37	419.08	119.06	175.26	12.03	57.83
12	9	46.0	-8.15	2.98	-4.99	139.16	139.31	365.80	364.06	64.71	153.45	8.67	52.83
12	10	9.0	-9.76	1.06	-6.98	144.08	143.81	355.71	354.29	61.23	149.60	9.82	47.94
12	11	31.0	-9.82	3.24	-2.43	148.20	148.11	343.79	342.60	59.48	145.28	10.34	43.12
12	12	54.0	-10.00	4.10	-1.32	151.06	150.81	329.78	326.51	57.14	140.94	10.80	38.23
13	46	30.0	12.35	-0.63	0.51	123.66	123.90	402.53	400.22	55.12	173.48	12.37	63.33
13	47	52.0	11.85	-0.58	-0.13	128.72	128.54	400.11	399.03	52.52	175.21	11.87	58.49
13	49	15.0	11.00	-0.63	-0.81	133.69	133.50	395.78	394.63	49.95	176.00	11.02	53.61
13	50	38.0	10.35	-0.66	-1.20	138.30	138.08	389.13	387.50	47.44	174.38	10.37	48.72
13	52	0.0	9.84	-0.70	-1.37	142.24	142.17	380.02	377.77	44.99	171.44	9.86	43.90
13	53	23.0	8.67	0.28	2.64	145.23	145.17	368.12	363.01	44.01	166.89	8.68	39.01
13	54	45.0	7.12	-0.80	-3.22	146.70	147.16	353.80	350.66	41.37	162.10	7.16	34.19
13	56	8.0	5.44	-0.66	-4.34	146.25	146.83	336.95	333.38	40.24	156.43	5.48	29.30
13	57	31.0	3.84	-0.47	-3.96	143.65	144.57	318.15	314.68	39.23	150.38	3.87	24.42
13	58	53.0	2.53	0.58	-2.87	139.06	140.29	298.24	293.51	39.28	144.97	2.60	19.59
15	26	59.0	-1.89	4.08	-0.80	119.17	119.86	414.73	414.60	68.80	171.11	4.50	64.10
15	28	21.0	-0.92	4.05	-1.08	123.87	123.18	413.88	414.97	65.31	170.79	4.15	59.27
15	29	44.0	1.14	3.77	0.97	128.45	128.28	410.55	411.58	60.11	170.27	3.94	54.38
15	31	6.0	3.17	3.66	2.32	132.61	132.48	404.33	404.37	55.61	169.53	4.84	49.56
15	32	29.0	4.92	3.63	1.43	136.18	136.08	394.74	394.15	51.70	168.07	6.11	44.67
15	33	52.0	6.50	3.41	1.90	138.74	138.83	381.78	380.88	48.09	166.02	7.33	39.79
15	35	14.0	7.82	3.32	3.98	139.85	139.09	365.89	360.57	45.71	163.39	8.49	34.96
15	36	37.0	9.48	3.11	3.89	139.17	138.38	347.10	341.43	43.29	160.38	9.98	30.08
15	37	59.0	10.35	2.84	3.96	136.51	136.91	326.55	320.70	41.21	156.27	10.72	25.26
15	39	22.0	11.56	2.18	4.33	131.80	133.45	304.62	300.67	39.39	152.03	11.76	20.37

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Table 5

Triad Attitude Determination Results, 12 September 1972

***** COMMENCE ATTITUDE DETERMINATION PROCESSING *****

EULER ANGLE SEQUENCE : POSITIVE PITCH MEANS NOSE DOWN
POSITIVE ROLL MEANS RIGHT WING DOWN
POSITIVE YAW MEANS NOSE LEFT

ORBIT PARAMETERS

EPOCH -TIME = 82.833 (KSECS-UT)	DAY OF EPOCH = 251	YEAR OF EPOCH = 1972
SEMI MAJOR AXIS= 1.123677 (EARTH RADII)	ARG. OF PERIGEE= 333.4968 (DEG)	PERIGEE ALT.= 401.31 (NAUT MI)
ECCENTRICITY= 0.00629	PRECESSION PER.= -3.30108 (DEG/DAY)	APOGEE ALT. = 449.98 (NAUT MI)
INCLINATION= 90.12863 (DEG)	RT. ASCENSION OF NODE AT EPOCH= 310.2959 (DEG)	
ORBIT PERIOD= 100.64 (MIN)	PRECESSION OF NODE= 0.01336 (DEG/DAY)	

STARTING ON DAY 256 AT UT TIME 2570.0 SECS

TIME HRS-MIN-SECS	ESTIMATED ATTITUDE ANGLES IN DEGREES PITCH	ROLL	YAW	ANG BET SUN AND GEOMAG VCTRS (DEG) THEORET	OBSERVD	GEOMAG VCTR MAGNITUDE (MOE) THEORET	OBSERVD	ANG BET VEH Z-AXIS AND FOLLOWING VECTORS IN DEGREES SUNLINE	GEOMAGN	LOCVERT	SATELLT LATITDE DEGREES
0 42 50.0	-7.57	2.55	1.67	49.58	50.76	382.03	377.46	121.44	171.76	7.98	51.19
4 3 47.0	2.36	0.31	1.64	56.92	58.87	410.15	410.16	113.69	163.69	2.38	49.57
4 5 10.0	3.08	0.61	1.46	61.74	63.09	416.44	414.07	110.07	165.90	3.14	54.51
12 46 50.0	-4.05	-0.06	-0.75	126.93	127.41	389.70	387.01	66.70	164.85	4.05	61.07
12 48 13.0	-2.96	0.28	-2.17	132.09	131.99	384.57	385.26	62.98	163.60	2.97	56.19
12 49 36.0	-1.81	0.21	-2.16	137.02	137.57	377.64	376.49	58.51	161.77	1.83	51.30
12 50 58.0	0.42	0.83	-0.91	141.88	141.36	365.74	366.74	55.00	160.41	0.94	46.48
12 52 21.0	0.81	0.52	-2.11	145.20	145.27	357.46	355.04	51.36	158.20	0.96	41.59
12 53 43.0	1.49	-0.15	-4.49	147.70	148.03	344.14	340.23	47.69	155.51	1.51	36.77
12 55 6.0	3.57	0.94	-2.02	148.50	148.35	328.62	324.42	45.42	153.21	3.69	31.89
12 56 29.0	4.30	0.79	-3.69	147.19	147.51	311.37	307.57	43.21	149.69	4.37	27.00
12 57 51.0	6.05	1.94	-1.43	143.75	145.10	293.07	290.08	42.25	146.41	6.35	22.17
14 25 57.0	-8.62	-2.73	-0.88	117.04	117.87	409.68	413.19	72.26	166.40	9.04	66.68
14 27 19.0	-6.58	-0.30	-0.14	121.82	122.11	409.59	409.01	69.02	166.45	6.59	61.85
14 28 42.0	-5.18	2.67	1.06	126.56	126.62	407.72	407.66	66.70	164.94	5.82	56.96
14 30 4.0	-5.30	2.37	1.41	131.00	130.73	403.54	404.09	63.41	162.62	5.81	52.14
14 31 27.0	-6.11	1.85	1.53	135.07	135.20	396.52	395.95	60.11	158.98	6.38	47.25
14 32 50.0	-6.48	1.74	1.07	138.41	137.62	386.45	383.68	57.54	155.66	6.71	42.37
14 34 12.0	-6.32	1.49	0.90	140.63	140.45	373.48	368.37	54.03	152.14	6.49	37.55
14 35 35.0	-5.68	1.97	1.80	141.38	140.64	357.46	352.82	51.52	148.74	6.01	32.66
14 36 57.0	-5.34	2.14	1.01	140.31	139.84	339.18	335.62	48.93	144.60	5.75	27.84
14 38 20.0	-5.15	1.37	0.69	137.25	138.22	318.78	315.49	45.60	139.99	5.33	22.95
14 39 43.0	-4.50	2.08	2.36	132.25	132.84	297.25	290.78	44.35	134.99	4.95	18.06
16 7 48.0	6.93	1.27	-2.42	118.06	117.42	414.40	413.75	61.61	178.97	7.04	62.63
16 9 11.0	5.97	1.14	-4.74	122.48	121.82	411.89	410.39	58.92	176.51	6.08	57.74
16 10 33.0	5.91	1.30	-1.53	126.61	126.04	406.60	404.09	55.98	173.65	6.05	52.92
16 11 56.0	4.97	1.60	-1.03	130.33	130.13	398.04	396.30	53.66	169.46	5.22	48.03
16 13 18.0	4.23	1.92	-4.06	133.28	132.48	386.26	382.26	51.92	165.21	4.65	43.21
16 14 41.0	3.95	2.05	-4.13	135.17	134.48	371.12	365.86	49.59	160.81	4.45	38.32
16 16 4.0	2.96	2.14	-2.21	135.59	135.70	353.16	349.66	47.50	155.29	3.65	33.44
16 17 26.0	2.05	2.57	-1.11	134.25	134.22	333.28	326.97	46.35	149.45	3.29	28.61

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Table 6

Triad Attitude Determination Results, 14 September 1972

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*****  COMMENCE ATTITUDE DETERMINATION PROCESSING  *****

EULER ANGLE SEQUENCE : POSITIVE PITCH MEANS NOSE DOWN
                       POSITIVE ROLL MEANS RIGHT WING DOWN
                       POSITIVE YAW MEANS NOSE LEFT

                         ORBIT PARAMETERS

EPOCH -TIME = 82.833 (KSECS-UT)          DAY OF EPOCH = 251                            YEAR OF EPOCH = 1972
SEMI MAJOR AXIS= 1.123677 (EARTH RADII)  ARG. OF PERIGEE= 333.4968 (DEG)               PERIGEE ALT.= 401.31 (NAUT MI)
ECCENTRICITY= 0.00629                    PRECESSION PER.= -3.30108 (DEG/DAY)           APOGEE ALT. = 449.98 (NAUT MI)
INCLINATION= 90.12863 (DEG)              RT. ASCENSION OF NODE AT EPOCH= 310.2959 (DEG)
ORBIT PERIOD= 100.64 (MIN)               PRECESSION OF NODE= 0.01336 (DEG/DAY)

                  STARTING ON DAY 258 AT UT TIME   4937.0 SECS
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TIME HRS-MIN-SECS	* ESTIMATED ATTITUDE ANGLES * IN DEGREES: PITCH	ROLL	YAW	* ANG BET SUN AND GEOMAG VCTRS (DEG): THEORET	OBSERVD	* GEOMAG VCTR MAGNITUDE (MOE): THEORET	OBSERVD	* ANG BET VEH Z-AXIS AND FOLLOWING VECTORS IN DEGREES: SUNLINE	GEOMAGN	LOCVERT	* SATELLT LATITDE DEGREES
1 22 17.0	4.88	2.07	1.02	49.06	50.68	390.66	388.61	114.45	160.16	5.30	47.87
1 23 39.0	4.75	1.63	2.35	54.15	55.49	398.02	394.73	110.91	163.06	5.02	52.75
1 25 2.0	5.05	1.84	2.82	59.45	60.62	402.89	398.51	107.36	165.40	5.38	57.69
1 26 24.0	5.22	1.37	2.09	64.71	67.19	405.53	402.42	102.70	168.03	5.39	62.55
3 4 8.0	-4.08	1.46	-2.83	58.12	58.62	414.11	413.26	117.88	172.40	4.34	51.97
3 5 31.0	-3.73	1.42	-2.03	62.97	63.56	417.92	419.36	114.17	174.82	3.99	56.90
3 6 53.0	-2.64	1.31	-1.95	67.82	69.10	418.95	418.53	109.54	176.28	2.95	61.77
13 27 40.0	0.13	0.34	-2.27	124.96	124.61	399.31	399.51	64.71	170.19	0.39	59.54
13 29 2.0	-0.64	0.67	-2.33	129.65	129.00	395.51	395.32	62.39	167.30	0.93	54.72
13 30 25.0	-0.91	0.77	-1.92	134.09	134.28	389.47	388.39	59.09	164.12	1.20	49.84
13 31 48.0	-1.57	0.93	-2.81	138.03	138.26	380.93	379.15	56.61	160.59	1.83	44.95
13 33 10.0	-1.72	1.41	-1.93	141.12	141.33	369.89	366.32	54.32	157.19	2.22	40.13
13 34 33.0	-2.64	1.35	-2.96	143.05	143.83	356.12	352.97	51.92	152.68	2.97	35.24
13 35 56.0	-4.26	0.97	-3.81	143.39	142.48	339.93	338.93	50.59	147.41	4.37	30.36
13 37 18.0	-4.38	0.74	-3.55	141.86	140.36	321.92	318.26	48.35	142.81	4.44	25.53
13 38 41.0	-4.31	1.91	-3.18	138.36	137.59	302.17	299.41	47.23	137.75	4.72	20.64
13 40 3.0	-4.73	1.59	-2.78	133.11	133.78	281.84	282.24	45.21	132.18	4.99	15.81
15 7 38.0	4.93	1.38	1.25	118.39	117.74	413.80	413.90	64.15	178.12	5.12	62.15
15 9 0.0	5.12	1.50	0.84	122.76	122.80	412.23	410.59	60.64	176.08	5.34	57.32
15 10 23.0	5.39	1.25	1.76	126.95	126.86	407.99	407.98	57.34	173.89	5.53	52.44
15 11 46.0	5.44	0.48	1.45	130.72	130.62	400.63	398.68	53.84	170.90	5.46	47.55
15 13 8.0	5.55	0.65	1.51	133.74	133.56	390.07	386.66	51.36	167.58	5.59	42.73
15 14 31.0	6.06	-3.51	-0.20	135.77	142.16	376.09	373.10	42.06	163.43	7.00	37.84

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Table 7

Triad Attitude Determination Results, 15 September 1972

***** COMMENCE ATTITUDE DETERMINATION PROCESSING *****

EULER ANGLE SEQUENCE : POSITIVE PITCH MEANS NOSE DOWN
POSITIVE ROLL MEANS RIGHT WING DOWN
POSITIVE YAW MEANS NOSE LEFT

ORBIT PARAMETERS

EPOCH -TIME = 82.833 (KSECS-UT)
SEMI MAJOR AXIS= 1.123677 (EARTH RADII)
ECCENTRICITY= 0.00629
INCLINATION= 90.12863 (DEG)
ORBIT PERIOD= 100.64 (MIN)

DAY OF EPOCH = 251
ARG. OF PERIGEE= 333.4968 (DEG)
PRECESSION PER.= -3.30108 (DEG/DAY)
RT. ASCENSION OF NODE AT EPOCH= 310.2959 (DEG)
PRECESSION OF NODE= 0.01336 (DEG/DAY)

YEAR OF EPOCH = 1972
PERIGEE ALT.= 401.31 (NAUT MI)
APOGEE ALT. = 449.98 (NAUT MI)

STARTING ON DAY 259 AT UT TIME 15248.0 SECS

TIME HRS-MIN-SECS	ESTIMATED ATTITUDE ANGLES IN DEGREES PITCH	ROLL	YAW	ANG BET SUN AND GEOMAG VCTRS (DEG) THEORET	OBSERVD	GEOMAG VCTR MAGNITUDE (MOE) THEORET	OBSERVD	ANG BET VEH Z-AXIS AND FOLLOWING VECTORS IN DEGREES SUNLINE	GEOMAGN	LOCVERT	SATELLIT LATITDE DEGREES
4 14 8.0	-6.20	1.78	96.53	58.79	58.56	406.58	407.77	121.03	170.02	6.45	50.04
4 15 31.0	-13.53	2.76	50.36	63.10	59.39	413.19	411.18	125.09	175.02	13.80	54.97
12 57 12.0	0.10	-0.82	21.23	124.31	124.47	394.12	391.79	65.05	169.83	0.84	60.66
12 58 34.0	6.55	-1.81	3.79	129.09	129.11	389.89	387.97	57.02	173.71	6.79	55.83
12 59 56.0	12.14	-4.12	-3.40	133.63	133.51	383.76	381.99	49.07	177.36	12.81	51.01
13 1 19.0	19.16	-4.71	17.29	137.79	137.45	375.32	373.95	42.83	177.40	19.71	46.13
13 2 42.0	24.85	-4.64	3.15	141.25	141.12	364.47	362.88	39.25	175.12	25.25	41.24
13 4 4.0	29.42	-5.73	-12.17	143.59	144.51	351.37	347.18	36.21	174.40	29.92	36.42
13 5 27.0	35.06	-6.36	14.16	144.48	144.45	335.88	332.24	36.64	172.82	35.56	31.53
13 6 49.0	39.13	-4.96	5.81	143.56	143.87	318.67	316.12	40.06	172.76	39.39	26.70
13 8 12.0	41.40	-5.25	-16.69	140.67	141.64	299.79	294.70	42.24	175.00	41.67	21.81
13 9 34.0	44.42	-6.10	-5.17	135.98	140.86	280.25	274.38	43.40	175.58	44.75	16.98
14 36 18.0	49.91	-5.22	-18.47	114.71	112.57	411.52	406.94	39.33	133.87	50.11	66.32
14 37 40.0	48.10	-4.74	-26.98	119.19	118.83	411.70	408.76	38.55	138.38	48.27	61.50
14 39 3.0	45.45	-4.21	-12.79	123.61	123.65	409.93	409.93	38.57	143.53	45.60	56.61
14 40 25.0	41.87	-2.96	0.28	127.74	128.24	405.66	403.91	39.48	149.94	41.96	51.79
14 41 48.0	37.06	-1.49	-8.57	131.48	131.17	398.36	395.38	41.26	157.60	37.09	46.90
14 43 10.0	32.01	-1.95	-30.75	134.48	134.56	387.98	384.94	40.65	166.03	32.06	42.08
14 44 33.0	27.24	-2.98	-43.56	136.48	136.37	374.27	371.76	39.71	173.81	27.39	37.19
14 45 56.0	22.87	-3.93	-39.34	137.11	137.87	357.60	355.02	38.28	174.77	23.19	32.31
14 47 18.0	17.38	-3.67	-27.77	136.07	136.71	338.68	335.70	38.68	165.96	17.76	27.48
14 48 41.0	11.18	-2.97	-24.05	133.18	134.15	317.72	315.14	39.45	154.98	11.56	22.59
14 50 3.0	4.22	-3.23	-35.24	128.51	130.20	296.09	294.53	39.42	142.48	5.31	17.75
16 19 31.0	-33.15	6.78	-40.78	120.12	119.97	405.56	415.56	91.88	136.53	33.76	57.45
16 20 54.0	-35.11	10.13	-21.15	123.78	123.67	403.11	403.32	92.00	131.95	36.36	52.56
16 22 17.0	-34.15	18.41	-23.17	127.21	130.97	393.66	397.54	91.91	127.43	38.26	47.68
16 23 39.0	-36.25	16.42	-41.91	129.90	130.42	381.11	386.76	90.04	123.52	39.33	42.86
16 25 2.0	-35.94	16.10	-27.11	131.61	130.62	365.35	367.40	86.79	120.34	38.93	37.97

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Table 8

Triad Attitude Determination Results, 22 September 1972

***** COMMENCE ATTITUDE DETERMINATION PROCESSING *****

EULER ANGLE SEQUENCE : POSITIVE PITCH MEANS NOSE DOWN
POSITIVE ROLL MEANS RIGHT WING DOWN
POSITIVE YAW MEANS NOSE LEFT

ORBIT PARAMETERS

EPOCH -TIME = 6.027 (KSECS-UT)	DAY OF EPOCH = 263	YEAR OF EPOCH = 1972
SEMI MAJOR AXIS= 1.123661 (EARTH RADII)	ARG. OF PERIGEE= 290.9424 (DEG)	PERIGEE ALT.= 403.28 (NAUT MI)
ECCENTRICITY= 0.00577	PRECESSION PER.= -3.40450 (DEG/DAY)	APOGEE ALT. = 447.90 (NAUT MI)
INCLINATION= 90.12721 (DEG)	RT. ASCENSION OF NODE AT EPOCH= 310.4585 (DEG)	
ORBIT PERIOD= 100.64 (MIN)	PRECESSION OF NODE= 0.01276 (DEG/DAY)	

STARTING ON DAY 266 AT UT TIME 2554.0 SECS

TIME HRS-MIN-SECS	ESTIMATED ATTITUDE ANGLES IN DEGREES PITCH	ROLL	YAW	ANG BET SUN AND GEOMAG VCTRS (DEG) THEORET	OBSERVD	GEOMAG VCTR MAGNITUDE (MOE) THEORET	OBSERVD	ANG BET VEH Z-AXIS AND POLLOWING VECTORS IN DEGREES SUNLINE	GEOMAGN	LOCVERT	SATELLT LATITDE DEGREES
0 42 34.0	5.23	-3.98	3.71	51.30	52.48	384.72	382.51	109.04	157.56	6.57	48.03
0 43 56.0	4.58	-4.37	2.28	55.39	56.79	392.39	387.03	106.19	160.82	6.33	52.88
2 23 3.0	-0.63	7.15	1.24	56.33	58.11	403.17	404.42	121.69	164.45	7.17	47.25
2 24 25.0	-0.20	7.18	2.81	59.88	61.08	410.07	412.49	118.89	166.80	7.18	52.10
2 25 48.0	0.52	6.91	1.83	63.79	65.52	413.99	413.70	114.98	168.68	6.93	57.01
12 47 57.0	-2.80	5.46	-5.72	119.36	120.38	398.38	397.45	75.51	163.75	6.13	59.58
12 49 20.0	-3.46	5.79	-5.18	123.61	124.38	394.60	394.70	73.42	161.16	6.74	54.68
12 50 42.0	-3.64	6.17	-4.27	127.55	127.89	388.73	388.02	71.32	158.72	7.16	49.85
12 52 5.0	-4.28	5.89	-3.56	131.13	132.67	380.32	380.63	68.21	155.20	7.28	44.95
12 53 27.0	-4.56	6.57	-2.15	134.04	134.25	369.43	368.13	67.10	152.21	8.00	40.11
12 54 50.0	-5.07	5.67	-2.30	136.09	137.03	355.80	353.59	63.89	148.54	7.60	35.20
12 56 13.0	-4.92	8.08	0.51	136.95	135.53	339.75	335.79	64.83	144.56	9.45	30.29
14 31 11.0	5.60	-3.41	-2.76	121.70	121.04	407.09	405.23	59.16	172.33	6.55	50.62
14 32 34.0	4.72	-2.43	-4.96	124.88	122.46	398.98	397.16	58.74	168.81	5.31	45.73
14 33 56.0	5.27	-2.64	-4.56	127.40	127.89	387.62	381.65	54.54	165.61	5.90	40.88
14 35 19.0	4.54	-2.51	-5.40	129.06	128.37	372.83	369.90	53.35	160.92	5.19	35.98
14 36 41.0	3.54	-1.82	-6.50	129.55	129.24	355.29	353.12	52.27	155.62	3.98	31.13
14 38 4.0	2.99	-0.97	-7.24	128.66	128.92	335.16	333.67	51.29	150.22	3.15	26.22
14 39 27.0	2.02	-0.58	-8.23	126.17	126.64	313.47	309.59	50.43	143.75	2.11	21.31
14 40 49.0	1.52	0.62	-7.09	122.06	122.44	291.48	285.91	50.59	137.15	1.65	16.45

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Table 9

Triad Attitude Determination Results, 26 September 1972

***** COMMENCE ATTITUDE DETERMINATION PROCESSING *****

EULER ANGLE SEQUENCE : POSITIVE PITCH MEANS NOSE DOWN
POSITIVE ROLL MEANS RIGHT WING DOWN
POSITIVE YAW MEANS NOSE LEFT

ORBIT PARAMETERS

EPOCH -TIME = 6.027 (KSECS-UT)	DAY OF EPOCH = 263	YEAR OF EPOCH = 1972
SEMI MAJOR AXIS= 1.123661 (EARTH RADII)	ARG. OF PERIGEE= 290.9424 (DEG)	PERIGEE ALT.= 403.28 (NAUT MI)
ECCENTRICITY= 0.00577	PRECESSION PER.= -3.40450 (DEG/DAY)	APOGEE ALT. = 447.90 (NAUT MI)
INCLINATION= 90.12721 (DEG)	RT. ASCENSION OF NODE AT EPOCH= 310.4585 (DEG)	
ORBIT PERIOD= 100.64 (MIN)	PRECESSION OF NODE= 0.01276 (DEG/DAY)	

STARTING ON DAY 270 AT UT TIME 1341.0 SECS

TIME	ESTIMATED ATTITUDE ANGLES IN DEGREES			ANG BET SUN AND GEOMAG VCTRS (DEG)		GEOMAG VCTR MAGNITUDE (MOE)		ANG BET VEH Z-AXIS AND FOLLOWING VECTORS IN DEGREES			SATELLIT LATITDE
HRS-MIN-SECS	PITCH	ROLL	YAW	THEORET	OBSERVD	THEORET	OBSERVD	SUNLINE	GEOMAGN	LOCVERT	DEGREES
0 22 21.0	-3.69	1.74	4.33	51.92	52.90	380.19	375.22	118.39	166.96	4.08	47.00
0 23 44.0	-3.99	1.19	4.21	55.47	56.84	388.69	387.28	115.51	170.16	4.17	51.90
0 25 6.0	-3.85	1.02	4.09	59.30	60.95	394.67	392.84	112.69	172.65	3.99	56.74
0 26 29.0	-4.28	0.26	5.46	63.37	64.38	398.58	395.61	110.00	174.32	4.29	61.63
2 2 50.0	0.04	3.18	-4.03	57.25	58.50	399.71	399.97	118.13	164.62	3.18	46.22
2 4 13.0	-0.35	3.68	-3.18	60.23	61.57	407.42	406.56	116.23	167.97	3.70	51.12
2 5 35.0	-0.88	4.28	-2.99	63.56	65.12	411.99	410.86	114.34	170.87	4.37	55.96
2 6 58.0	-1.49	4.30	-3.48	67.17	67.81	413.93	412.23	112.44	173.56	4.55	60.86
12 27 45.0	1.07	2.14	-4.75	115.63	116.16	398.70	400.68	74.32	169.26	2.40	60.58
12 29 7.0	1.72	2.03	-3.95	119.63	120.17	395.33	394.39	71.22	167.90	2.66	55.74
12 30 30.0	2.44	2.80	-3.82	123.48	123.86	389.86	389.41	69.03	165.86	3.72	50.83
12 31 52.0	3.09	2.75	-3.66	126.94	128.29	382.08	381.15	65.84	163.54	4.14	45.99
12 33 15.0	3.13	2.81	-3.55	129.95	131.27	371.60	370.33	63.68	160.73	4.21	41.08
12 34 38.0	3.10	4.07	-2.86	132.24	131.97	358.50	357.02	63.58	157.42	5.12	36.16
12 36 0.0	3.53	4.09	-2.24	133.55	133.66	343.11	339.78	61.47	154.07	5.40	31.31
12 37 23.0	4.45	4.86	-2.05	133.65	133.68	325.41	323.37	60.42	150.55	6.59	26.38
12 38 45.0	4.65	4.72	-0.61	132.35	133.96	306.27	300.72	58.39	146.12	6.62	21.52
12 40 8.0	4.19	4.52	-0.06	129.49	130.93	285.88	281.39	57.42	140.64	6.16	16.59
14 7 51.0	-2.24	0.95	0.77	109.58	106.88	414.55	413.26	78.05	172.39	2.44	62.71
14 8 13.0	-3.05	0.06	2.96	110.56	111.17	414.56	415.87	75.39	170.06	3.05	61.41
14 9 36.0	-2.44	-0.27	2.61	114.20	114.40	413.21	414.06	72.17	168.43	2.45	56.51
14 10 59.0	-2.24	-0.52	3.19	117.65	118.01	409.14	410.09	69.08	165.84	2.31	51.61
14 12 21.0	-2.09	-1.32	2.08	120.73	121.23	401.99	399.33	65.68	162.86	2.47	46.77
14 13 44.0	-1.89	-0.55	1.28	123.33	122.71	391.36	389.62	64.30	159.75	1.97	41.86
14 15 6.0	-1.25	-1.15	0.46	125.18	125.07	377.54	372.71	60.95	156.41	1.70	37.00
14 16 29.0	-0.55	-0.86	1.23	126.11	126.12	360.45	357.33	58.74	152.74	1.03	32.09
14 17 52.0	-0.31	-0.34	-0.23	125.84	125.34	340.78	336.29	57.46	148.12	0.47	27.16
14 19 14.0	0.72	-0.39	-1.54	124.21	124.13	319.56	314.36	55.32	143.76	0.83	22.30
14 20 37.0	0.73	1.03	-0.48	121.02	121.07	297.23	293.78	55.33	137.78	1.27	17.37

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Table 10

Triad Attitude Determination Results, 27 September 1972

***** COMMENCE ATTITUDE DETERMINATION PROCESSING *****

EULER ANGLE SEQUENCE : POSITIVE PITCH MEANS NOSE DOWN
POSITIVE ROLL MEANS RIGHT WING DOWN
POSITIVE YAW MEANS NOSE LEFT

ORBIT PARAMETERS

EPOCH -TIME = 6.027 (KSECS-UT)	DAY OF EPOCH = 263	YEAR OF EPOCH = 1972
SEMI MAJOR AXIS= 1.123661 (EARTH RADII)	ARG. OF PERIGEE= 290.9424 (DEG)	PERIGEE ALT.= 403.28 (NAUT MI)
ECCENTRICITY= 0.00577	PRECESSION PER.= -3.40450 (DEG/DAY)	APOGEE ALT. = 447.90 (NAUT MI)
INCLINATION= 90.12721 (DEG)	RT. ASCENSION OF NODE AT EPOCH= 310.4585 (DEG)	
ORBIT PERIOD= 100.64 (MIN)	PRECESSION OF NODE= 0.01276 (DEG/DAY)	

STARTING ON DAY 271 AT UT TIME 5622.0 SECS

TIME	ESTIMATED ATTITUDE ANGLES IN DEGREES			ANG BET SUN AND GEOMAG VCTRS (DEG)		GEOMAG VCTR MAGNITUDE (MOE)		ANG BET VEH Z-AXIS AND FOLLOWING VECTORS IN DEGREES			SATELLT LATITDE
HRS-MIN-SECS	PITCH	ROLL	YAW	THEORET	OBSERVD	THEORET	OBSERVD	SUNLINE	GEOMAGN	LOCVERT	DEGREES
1 33 42.0	0.04	2.17	4.25	58.41	59.20	402.52	399.53	115.51	167.20	2.18	49.84
1 35 4.0	-0.46	1.99	4.54	61.64	63.37	407.94	406.97	112.69	170.69	2.04	54.68
1 36 27.0	-0.48	1.57	3.89	65.19	66.72	410.76	408.52	109.86	173.21	1.64	59.58
3 12 48.0	0.09	1.77	-3.40	60.17	62.59	391.64	391.05	117.09	160.93	1.78	44.16
3 14 11.0	0.47	2.20	-4.01	62.52	64.47	402.05	401.44	115.16	163.87	2.25	49.06
11 57 14.0	0.45	3.66	2.57	114.88	116.02	393.78	396.17	77.07	166.76	3.69	61.85
11 58 36.0	1.18	3.23	2.34	118.97	119.75	389.86	389.41	73.87	165.86	3.44	57.01
11 59 59.0	1.62	2.88	3.31	122.94	123.77	384.09	384.13	70.78	164.19	3.31	52.11
12 1 21.0	2.01	2.33	3.34	126.60	127.79	376.24	375.74	67.54	162.12	3.09	47.26
12 2 44.0	2.35	2.31	3.61	129.87	130.66	365.92	364.29	65.30	159.65	3.29	42.35
12 4 7.0	2.38	2.51	2.95	132.52	132.02	353.18	350.24	63.98	156.68	3.46	37.43
12 5 29.0	2.90	2.49	3.64	134.27	133.98	338.34	337.35	61.78	153.38	3.83	32.57
12 6 52.0	3.30	2.22	4.39	134.91	134.89	321.31	317.13	59.61	149.61	3.98	27.65
12 8 14.0	3.87	1.77	4.02	134.19	134.70	302.95	297.49	57.42	145.65	4.26	22.78
13 36 21.0	-2.62	0.37	-2.87	106.21	106.61	411.38	409.20	79.67	172.46	2.65	67.47
13 37 42.0	-2.15	0.69	-3.69	109.90	111.10	412.26	411.05	76.50	170.70	2.26	62.69
13 39 5.0	-2.18	0.66	-4.43	113.61	113.64	411.46	413.23	74.32	169.01	2.28	57.79
13 40 28.0	-1.55	0.98	-3.72	117.15	117.93	408.27	408.39	71.22	166.82	1.84	52.89
13 41 50.0	-1.53	1.20	-4.06	120.37	120.85	402.29	400.93	69.03	164.12	1.94	48.04
13 43 13.0	-1.68	1.58	-4.32	123.19	124.03	393.04	391.72	66.86	160.65	2.31	43.13
13 44 35.0	-1.51	1.83	-4.32	125.36	126.34	380.68	379.01	64.70	157.27	2.37	38.27
13 45 58.0	-1.00	2.69	-3.60	126.70	126.81	365.04	362.82	63.58	153.85	2.87	33.36
13 48 43.0	-0.22	3.49	-3.54	125.94	126.48	326.35	323.51	60.42	145.17	3.50	23.56
13 50 6.0	0.63	4.59	-1.66	123.46	125.50	304.49	302.96	59.41	140.45	4.64	18.63
15 26 27.0	0.11	-0.10	1.91	121.35	121.53	358.25	353.88	60.86	152.39	0.19	34.14
23 21 19.0	-2.18	3.53	-1.87	48.13	51.55	355.63	351.48	119.11	161.70	4.15	44.45
23 22 42.0	-2.25	4.03	-2.65	51.52	53.04	367.15	361.90	118.16	165.01	4.62	49.34
23 24 4.0	-2.74	3.47	-1.46	55.23	57.20	376.28	372.85	115.28	168.63	4.42	54.18

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Table 11

Triad Attitude Determination Results, 28 September 1972

***** COMMENCE ATTITUDE DETERMINATION PROCESSING *****

EULER ANGLE SEQUENCE : POSITIVE PITCH MEANS NOSE DOWN
POSITIVE ROLL MEANS RIGHT WING DOWN
POSITIVE YAW MEANS NOSE LEFT

ORBIT PARAMETERS

EPOCH -TIME = 6.027 (KSECS-UT)
SEMI MAJOR AXIS= 1.123661 (EARTH RADII)
ECCENTRICITY= 0.00577
INCLINATION= 90.12721 (DEG)
ORBIT PERIOD= 100.64 (MIN)

DAY OF EPOCH = 263
ARG. OF PERIGEE= 290.9424 (DEG)
PRECESSION PER.= -3.40450 (DEG/DAY)
RT. ASCENSION OF NODE AT EPOCH= 310.4585 (DEG)
PRECESSION OF NODE= 0.01276 (DEG/DAY)

YEAR OF EPOCH = 1972
PERIGEE ALT.= 403.28 (NAUT MI)
APOGEE ALT. = 447.90 (NAUT MI)

STARTING ON DAY 272 AT UT TIME 3708.0 SECS

TIME HRS-MIN-SECS	ESTIMATED ATTITUDE ANGLES IN DEGREES PITCH	ROLL	YAW	ANG BET SUN AND GEOMAG VCTRS (DEG) THEORET	OBSERVD	GEOMAG VCTR MAGNITUDE (MOE) THEORET	OBSERVD	ANG BET VEH Z-AXIS AND FOLLOWING VECTORS IN DEGREES SUNLINE	GEOMAGN	LOCVERT	SATELLT LATITDE DEGREES
1 1 48.0	0.81	-0.47	-1.28	53.85	55.31	386.11	384.53	115.13	161.92	0.94	43.67
1 3 11.0	0.14	-0.47	-1.29	56.55	58.00	395.55	393.74	113.23	165.73	0.51	48.56
1 4 33.0	-0.15	-0.12	-1.27	59.68	61.19	402.05	401.14	111.35	168.92	0.24	53.40
1 5 56.0	-0.62	-0.21	-1.91	63.16	63.87	406.00	402.02	109.46	171.53	0.67	58.30
1 7 19.0	-1.04	-0.33	-2.27	66.84	68.37	407.74	403.61	106.62	174.29	1.09	63.19
2 42 17.0	-0.13	3.07	2.68	59.17	60.31	391.39	391.07	119.23	160.92	3.07	42.89
2 43 40.0	0.10	2.62	3.31	61.32	62.81	402.11	400.56	116.35	164.31	2.62	47.79
2 45 2.0	-0.04	2.90	2.24	63.96	65.25	409.43	407.52	114.43	167.35	2.90	52.62
2 46 25.0	0.16	2.52	2.39	66.99	68.08	413.73	411.91	111.61	170.00	2.53	57.52
11 28 6.0	-0.07	-0.49	0.78	118.16	119.07	384.41	384.46	72.76	166.32	0.51	58.23
11 29 28.0	1.05	-0.48	0.65	122.18	123.24	378.18	378.92	69.54	164.73	1.16	53.38
11 36 21.0	2.78	0.35	-1.56	135.64	135.52	315.54	311.12	59.43	148.59	2.80	28.92
13 7 11.0	-1.29	3.58	1.01	109.29	110.37	408.94	408.47	79.88	169.48	3.81	63.97
13 8 34.0	-1.67	3.86	1.25	113.07	113.81	408.24	407.26	77.81	167.51	4.21	59.07
13 9 57.0	-1.81	3.07	0.67	116.72	117.42	405.50	406.12	74.62	165.77	3.56	54.16
13 11 19.0	-1.13	2.96	1.59	120.08	121.15	400.26	401.45	71.53	163.87	3.17	49.31
13 12 42.0	-1.26	2.76	1.38	123.11	123.63	392.05	390.43	69.31	161.20	3.04	44.40
13 14 4.0	-0.98	2.93	2.61	125.57	125.91	380.90	380.59	67.20	158.27	3.09	39.54
13 15 27.0	-0.98	2.17	2.51	127.31	128.66	366.60	364.91	63.98	154.60	2.39	34.62
13 16 50.0	-0.58	2.48	2.17	128.07	128.27	349.50	344.80	62.72	151.01	2.55	29.70
13 18 12.0	0.06	3.12	3.15	127.64	127.53	330.33	326.24	61.59	146.98	3.12	24.83
13 19 35.0	0.24	2.79	3.49	125.84	126.67	309.30	306.66	59.46	142.00	2.80	19.89
14 49 3.0	1.30	0.01	-2.43	108.41	109.09	416.17	416.59	73.42	173.22	1.31	59.85
14 50 25.0	1.39	0.36	-2.94	111.71	112.12	413.29	411.01	71.22	170.74	1.44	55.00
14 51 48.0	1.10	0.56	-3.43	114.83	115.27	407.25	407.44	69.03	167.40	1.24	50.09
14 53 11.0	0.83	0.68	-3.79	117.58	118.10	397.74	397.58	66.86	163.69	1.08	45.18
14 54 33.0	0.71	0.54	-3.94	119.77	120.02	384.87	381.11	64.70	159.77	0.90	40.33
14 55 56.0	0.70	1.49	-3.68	121.24	121.27	368.54	366.48	63.58	155.58	1.65	35.40
14 57 18.0	0.59	2.16	-3.33	121.74	121.47	349.67	345.55	62.49	150.79	2.24	30.54
14 58 41.0	0.57	2.09	-3.25	121.04	121.64	328.55	327.35	60.42	145.44	2.17	25.61

vectors. The other is the difference between the theoretical and observed geomagnetic field magnitudes. In general, the smaller the differences, the more accurate the attitude angle estimates. The observed and theoretical data are listed in columns 7 through 10 (from left to right) in each table.

RESULTS FROM TIP-II

The second satellite, TIP-II (see Fig. 6), was launched in 1975. It failed to achieve the desired structural configuration (for gravity-gradient stabilization) because of a malfunction of its extendible boom. The resulting moment-of-inertia distribution was unfavorable in that attitude stabilization about the desired spacecraft X-, Y-, and Z-axes was not possible.

The following attitude estimation results on 23 September 1976 were typical of the satellite's dynamics for more than 2 months. On this day, at 3 p.m. EST, approximately 30 min of attitude sensor data were processed. A summary of the attitude estimation results includes the following.

1. A total of 259 sets of digitized attitude sensor data was received before the satellite entered the night portion of the orbit. More than 80% of the sets passed all validity checks. Figure 7 is a plot of the sun sensor data. The ψ angle (ordinate axis) is the angle between the sunline vector and the vehicle Z-axis. The azimuth angle (abscissa axis) is the angle between the X-Y component of the sunline vector and the vehicle's X-axis. The time history of the magnetometer data is shown in Fig. 8.

2. The excellent agreement between the computed (theoretical) and measured (observed) magnetic field and sun data is shown in Figs. 9 and 10.

3. The satellite attitude dynamics was determined to be a combination of a 3- to 5-revolution per orbit pitch-axis tumble and a relatively rapid coning of the pitch axis in inertial space. Gravity-gradient stabilization of the reference Z-axis, as indicated by the pitch tumble, had not occurred. Indeed the dynamics appeared more representative of a spin-stabilized satellite. Figures 11 to 15 are plots of the estimated attitude angles. The last plot shows clearly the coning motion of the spacecraft's Y-axis in inertial space. The declination and right ascension angles are referenced to the geocentric reference system of axes (Z is the North Pole, X is the first point of Aries, and Y is the vector-completing right-hand set).

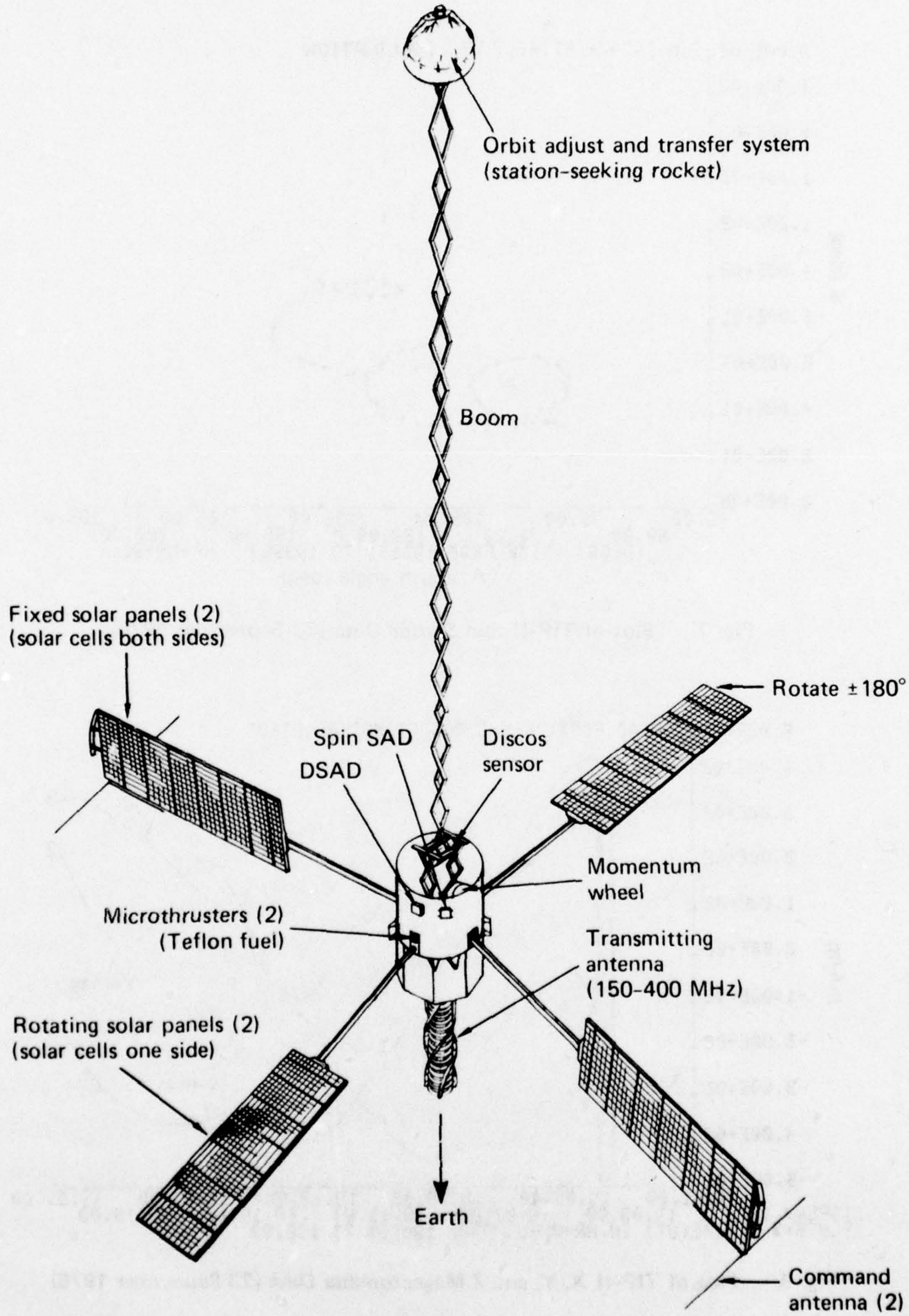

Fig. 6 Orbital Configuration of TIP-II and -III

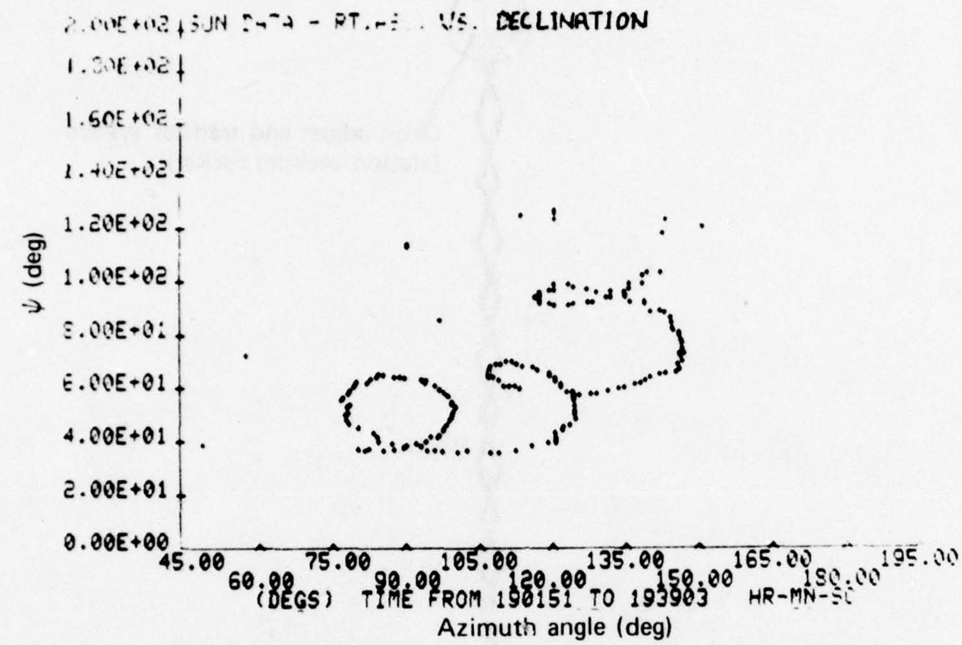

Fig. 7 Plot of TIP-II Sun Sensor Data (23 September 1976)

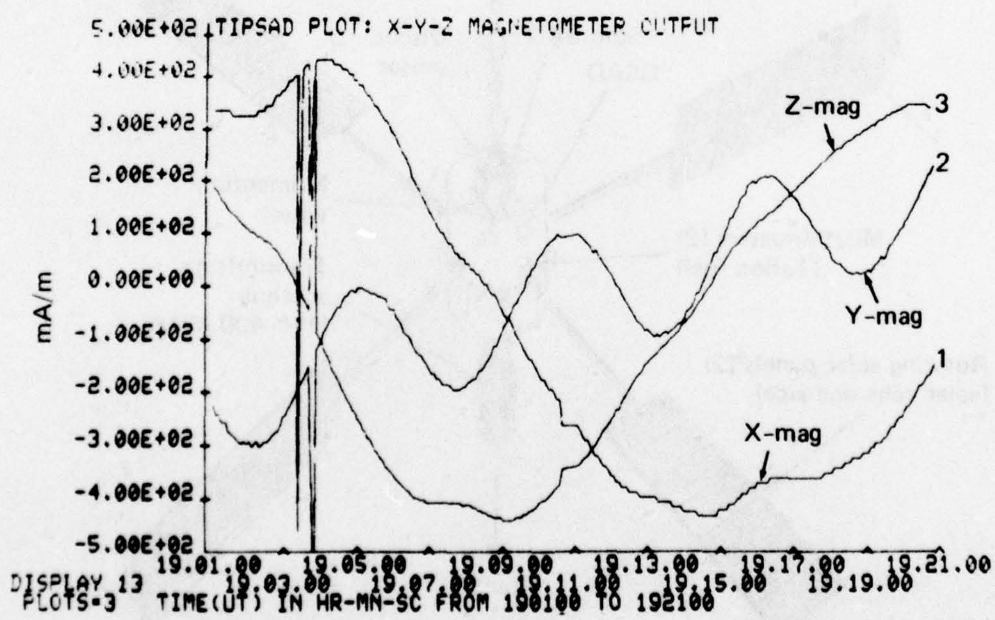

Fig. 8 Plot of TIP-II X, Y, and Z Magnetometer Data (23 September 1976)

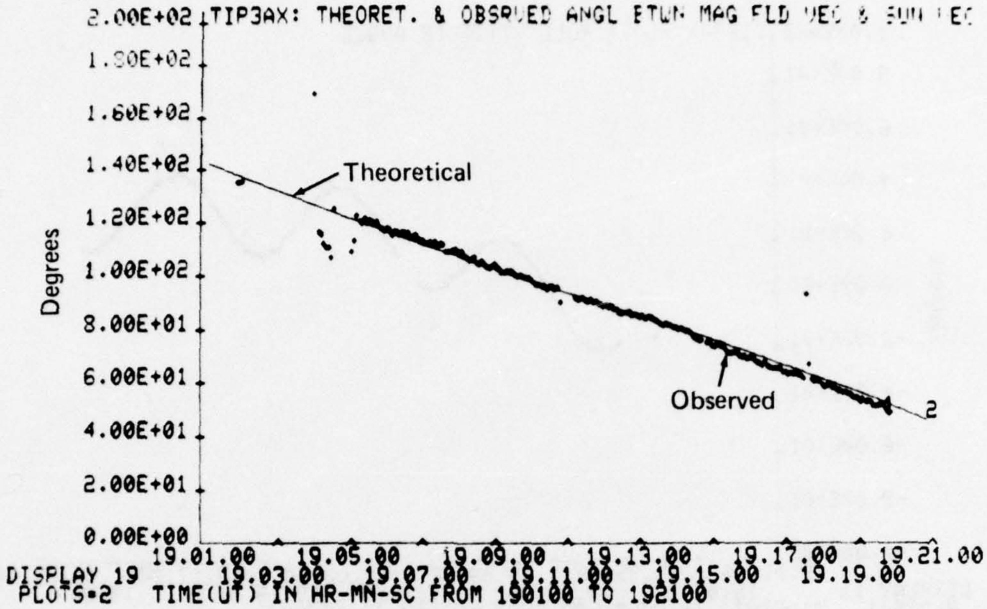

Fig. 9 Plot of Theoretical and Observed Angles Between Geomagnetic Field Vector and Sun Vector (TIP-II TLM, 23 September 1976)

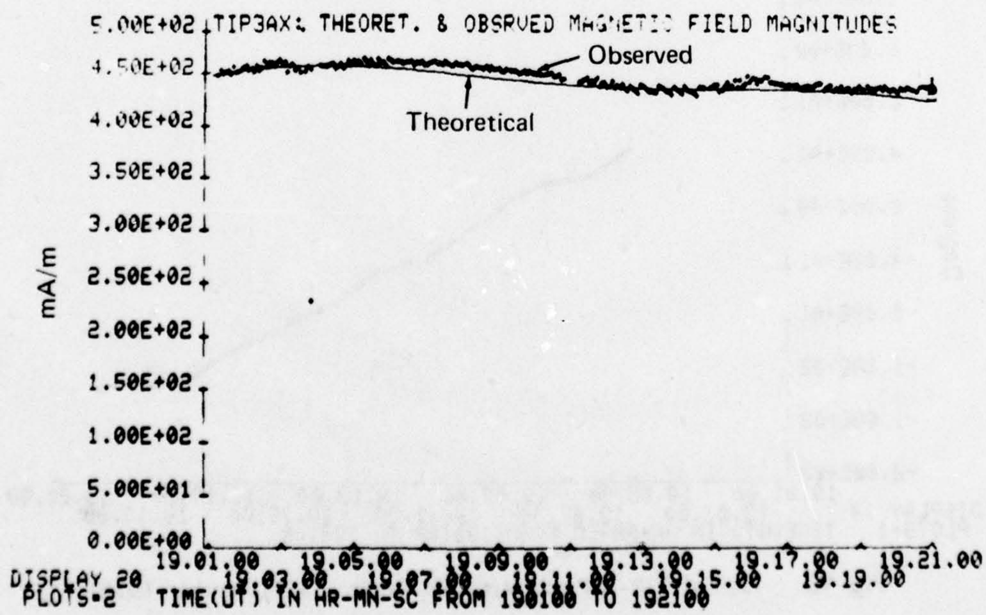

Fig. 10 Plot of Theoretical and Observed Magnetic Field Magnitude (TIP-II TLM, 23 September 1976)

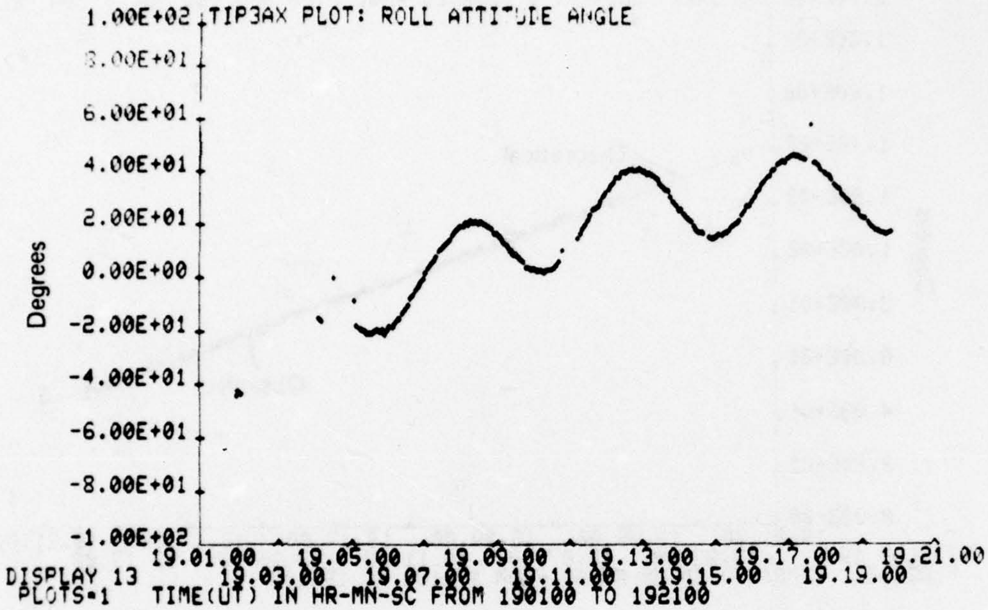

Fig. 11 Plot of TIP-II Roll Attitude Angle (23 September 1976)

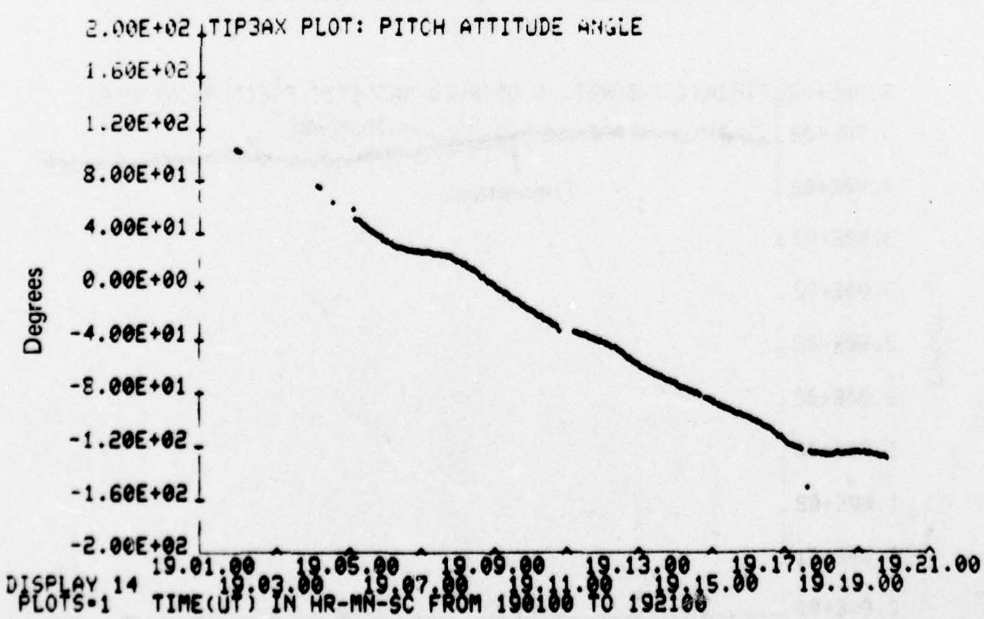

Fig. 12 Plot of TIP-II Pitch Attitude Angle (23 September 1976)

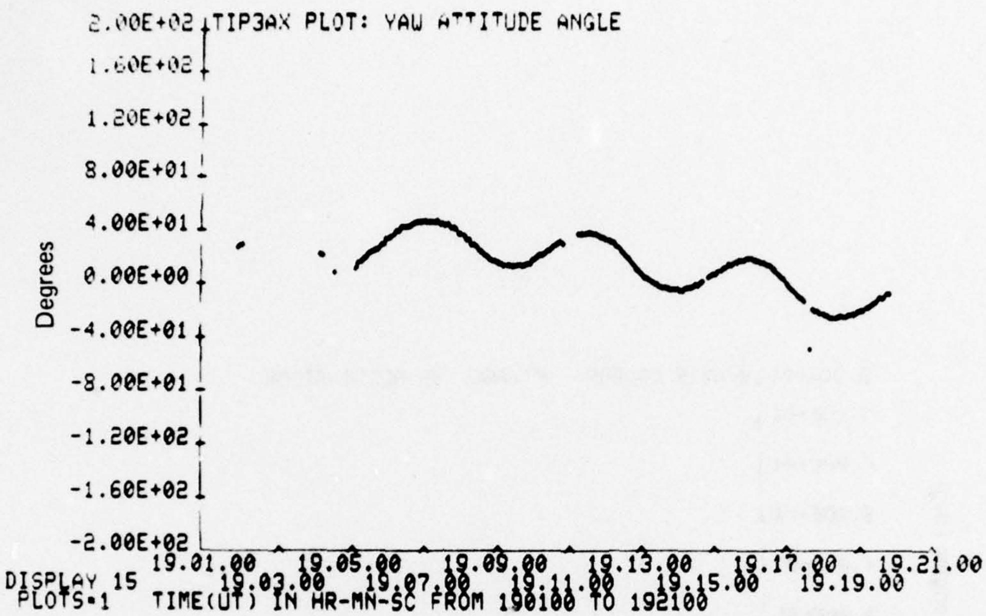

Fig. 13 Plot of TIP-II Yaw Attitude Angle (23 September 1976)

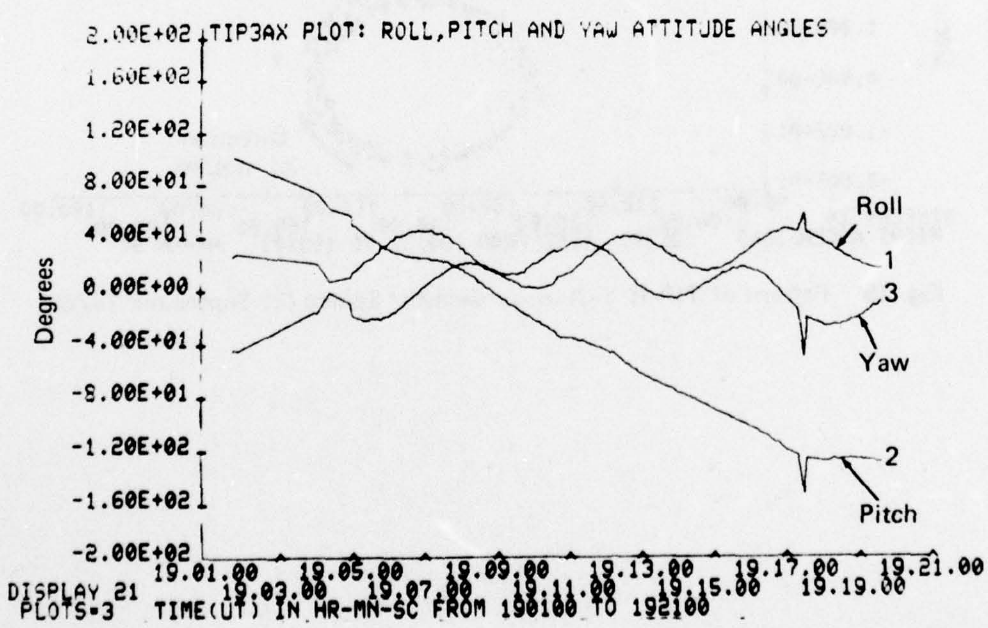

Fig. 14 Plot of TIP-II Roll, Pitch, and Yaw Attitude Angles (23 September 1976)

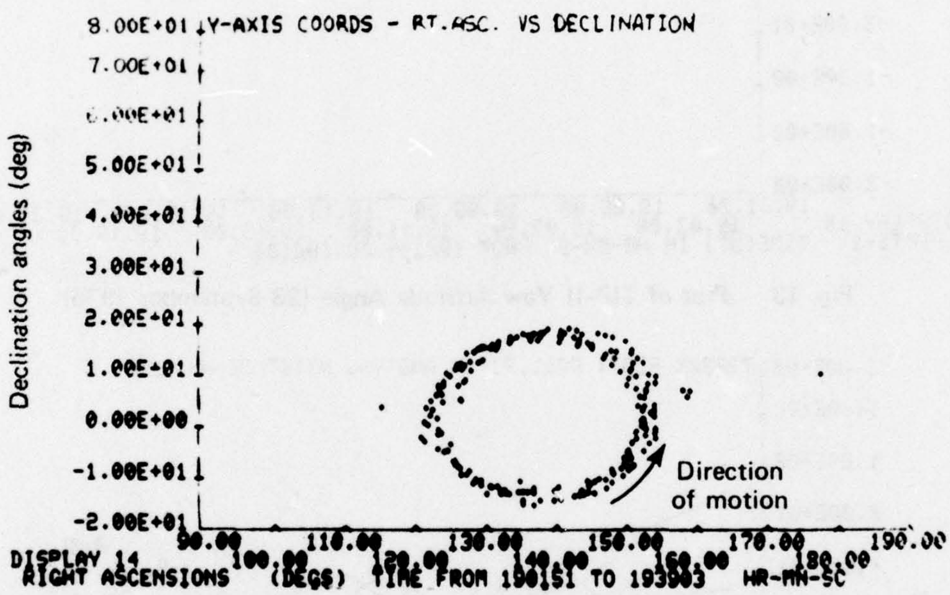

Fig. 15 Pattern of TIP-II Y-Axis on Celestial Sphere (23 September 1976)

RESULTS FROM TIP-III

TIP-III (the same configuration as TIP-II) was more successful than its twin in achieving gravity-gradient stabilization. During the early part of March 1977, the satellite's boom was successfully extended, and a right-side-up stabilization was achieved. The results presented here are from 10 March 1977 (12 noon), when the attitude was monitored over a period of three successive orbits. The results included the following.

1. The sun sensor data are displayed as time history plots in Figs. 16 and 17. The ψ and azimuth angles are as defined previously. The noon (ψ is close to zero) and midnight (ψ is approximately 180°) portions of each orbit are readily discernible in the figures.

2. The magnetometer data are shown in Fig. 18. The Z-magnetometer shows the approximate times the spacecraft crossed the North and South Poles with its minimum (-450 mA/m) and maximum (+450 mA/m) readings, respectively.

3. The peak roll angle was 30° (Fig. 19). The frequency of roll oscillation was approximately 2 revolutions per orbit.

4. The peak pitch angle was 15° (Fig. 20). The frequency of pitch libration was approximately 1.7 revolution per orbit.

5. During this time, the constant speed rotor was off. The satellite's inertia ellipsoid was very much like that of a dumbbell. Without the rotor, the moment-of-inertia distribution would not tend to stabilize very well (if at all) with the Y-axis aligned with the orbit normal. Thus it was not surprising when the spacecraft, on this day, was observed performing 360° rotations in yaw approximately once every 3 h (Fig. 21). The rotation was negative.

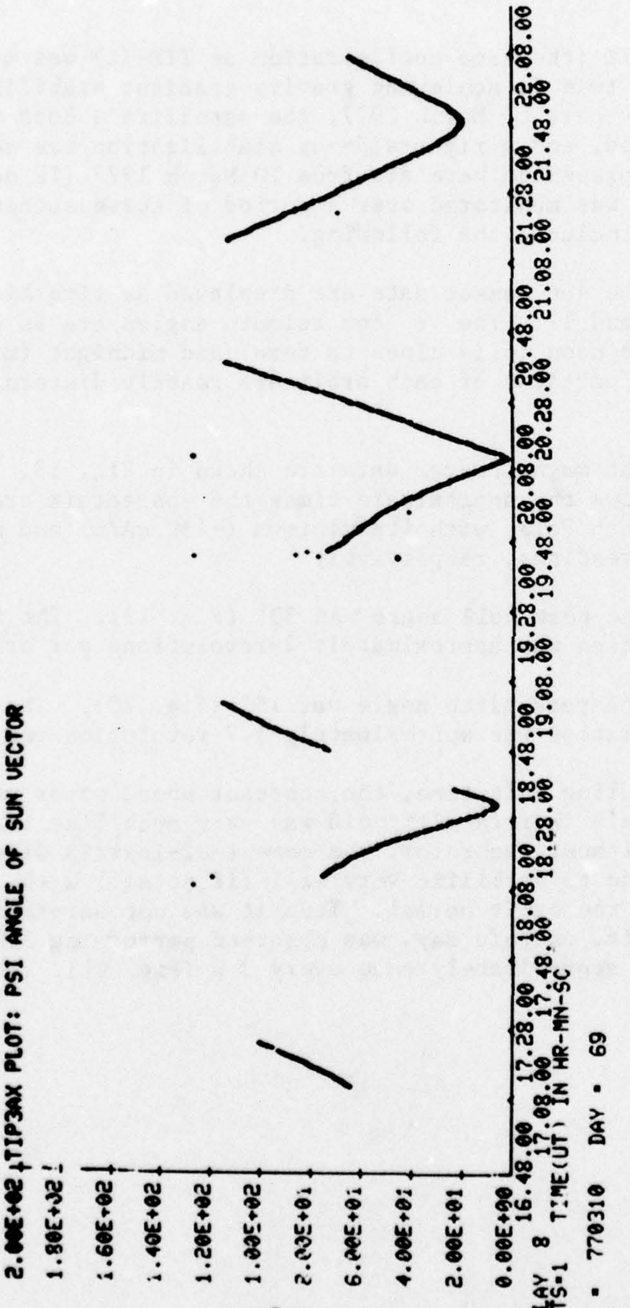

Fig. 16 Plot of Sun ψ Angle (TIP-III TLM, 10 March 1977)

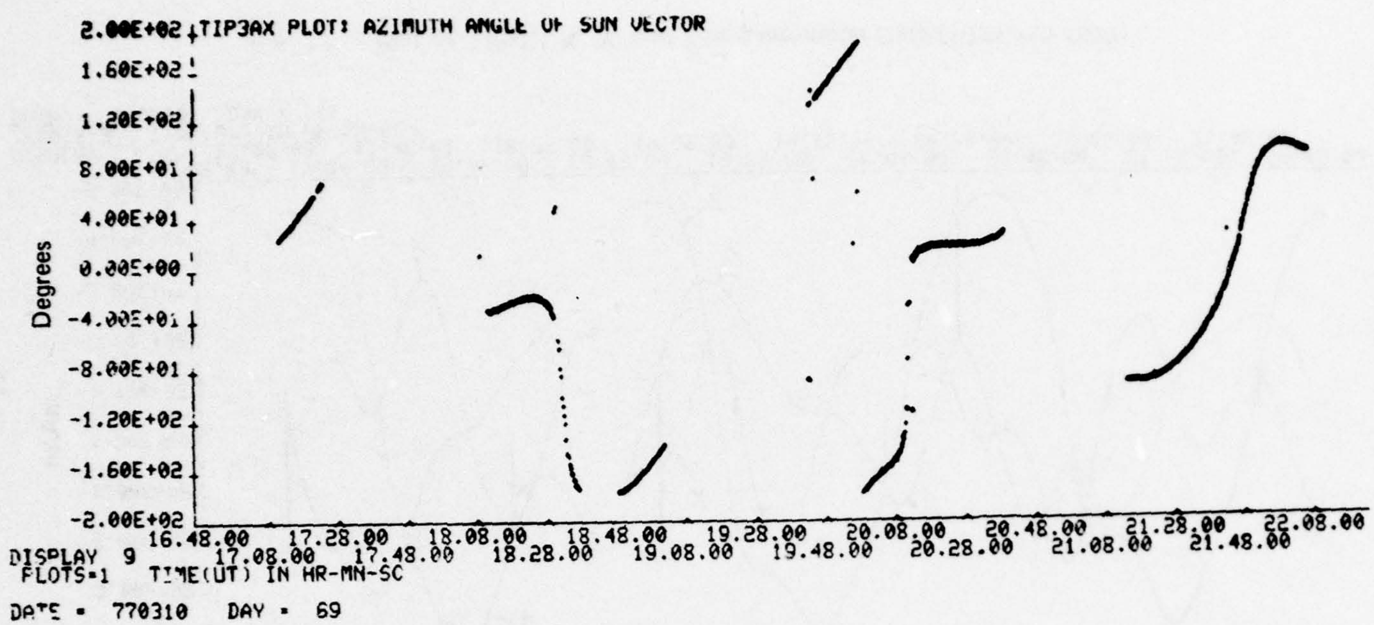

Fig. 17 Plot of Sun Azimuth Angle (TIP-III TLM, 10 March 1977)

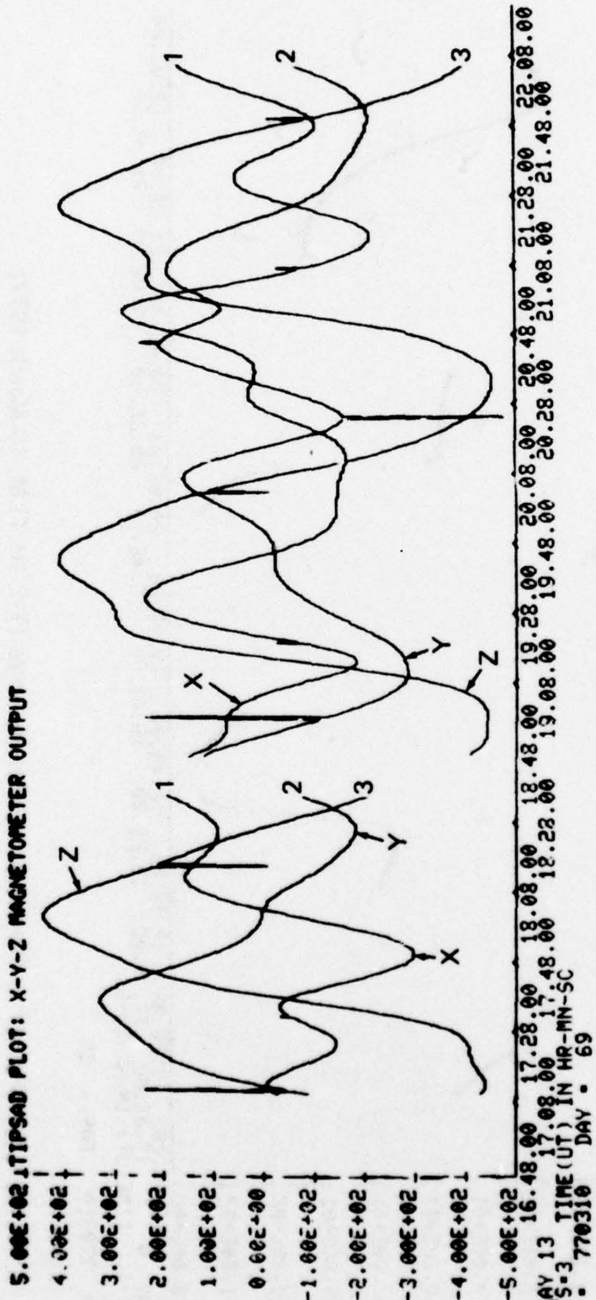

Fig. 18 Plot of TIP-III X, Y, and Z Magnetometer Data (10 March 1977)

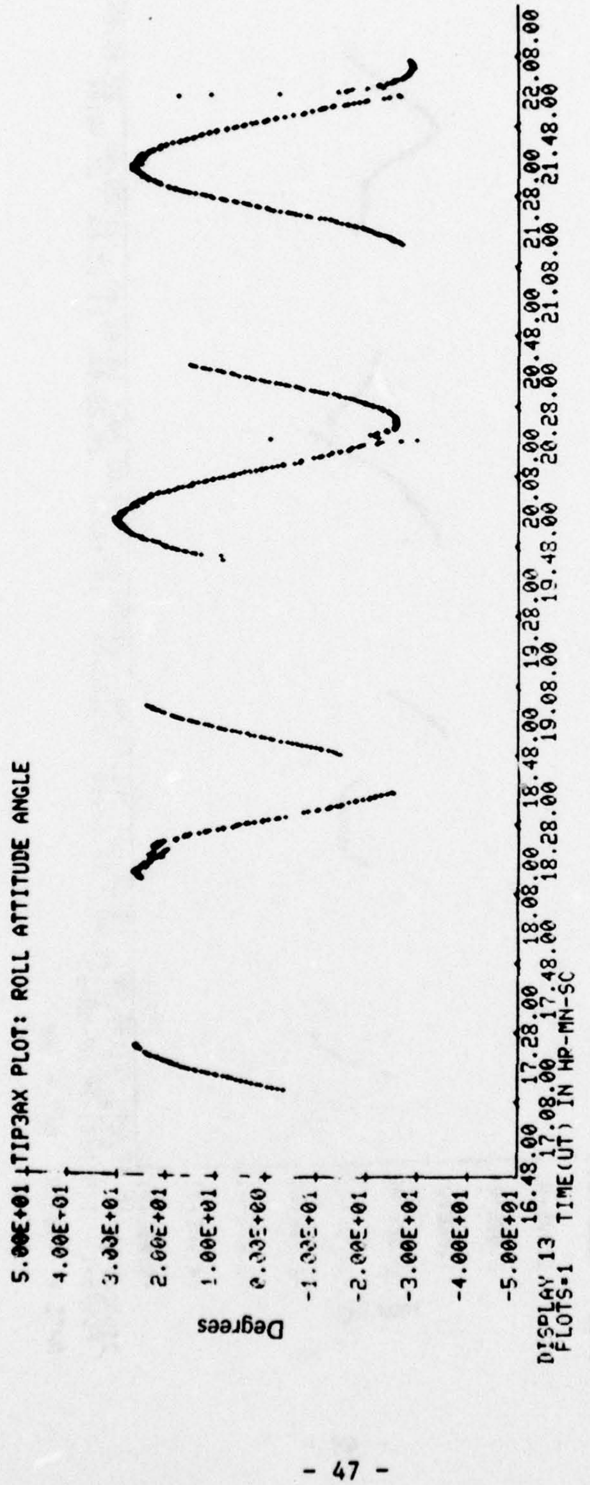

Fig. 19 TIP-III Roll Attitude Angle (10 March 1977)

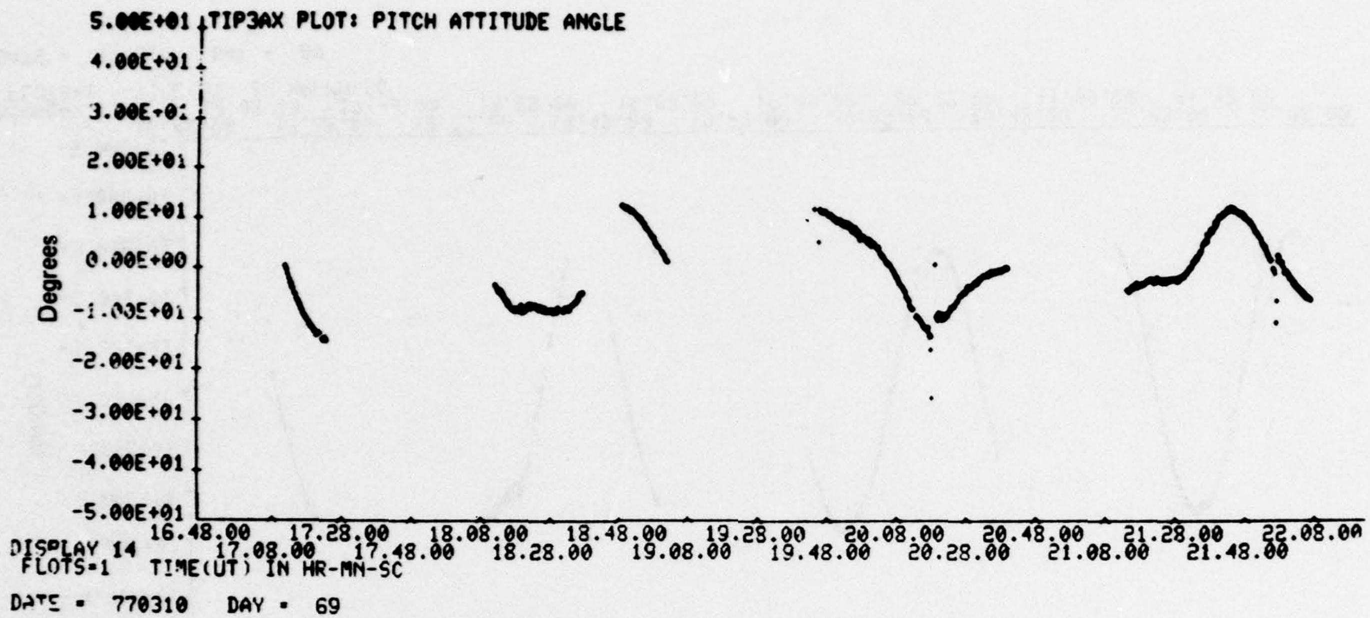

Fig. 20 TIP-III Pitch Attitude Angle (10 March 1977)

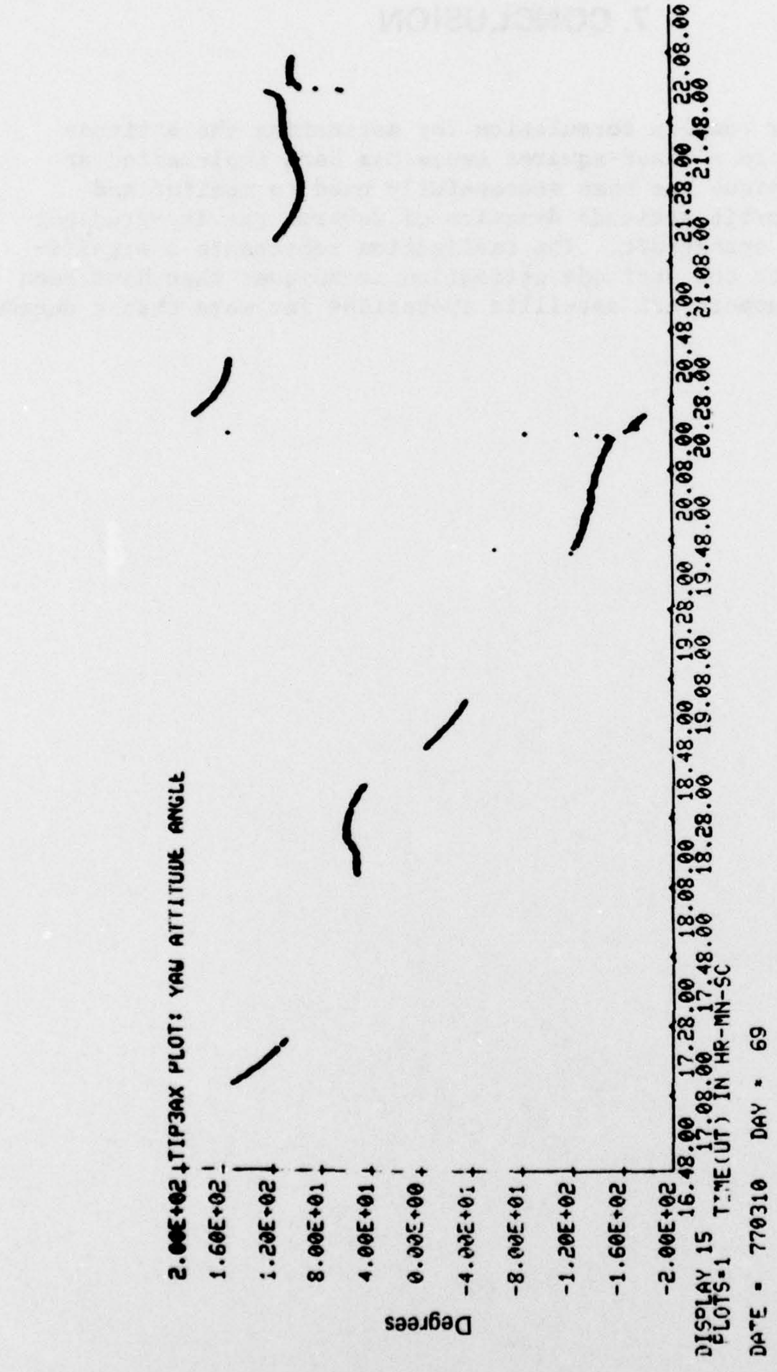

Fig. 21 TIP-III Yaw Attitude Angle (10 March 1977)

7. CONCLUSION

A rather complex formulation for estimating the attitude of a satellite in a least-squares sense has been implemented at APL. The technique has been successfully used to monitor and assess the in-orbit attitude dynamics of several gravity-gradient-stabilized APL spacecraft. Its realization represents a significant addition to the attitude estimation techniques that have been developed to support APL satellite operations for more than a decade.

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Appendix A
DERIVATION OF LEAST-SQUARES SOLUTION

In this Appendix, the least-squares solution to the attitude estimation problem will be derived. The discussion will parallel the development in Ref. 7. Capital alphabets will denote matrices, and small alphabets will denote column vectors. Scalars will be appropriately identified.

Given two sets of n unit vectors $m_1, m_2, \ldots, m_n$ and $m_1^*, m_2^*, \ldots, m_n^*$, where $n \geq 2$, find the rotation matrix A (i.e., the orthogonal matrix with determinant +1) that brings the first set into the best least-squares coincidence with the second. That is, find A, which minimizes

$$\sum_{j=1}^{n} \left|\left| m_j^* - Am_j \right|\right|^2 .$$

The problem has arisen in the estimation of the attitude of a satellite by using the unit vectors (m^*) of objects as observed in a satellite fixed reference system and the unit vectors (m) of the same objects in a known reference system. A is then a least-squares estimate of the rotation matrix that carries the known reference system into the satellite fixed reference system.

Let k denote the number of elements of the column vectors $m_1, \ldots, m_n, m_1^*, \ldots, m_n^*$ and let M and M^* denote the two k by n matrices whose columns are $m_1, \ldots, m_n$ and $m_1^*, \ldots, m_n^*$, respectively.

Now define Q(A) as the sum of squares to be minimized, i.e.,

$$Q(A) = \sum_{j=1}^{n} \left|\left| m_j^* - Am_j \right|\right|^2 = \mathrm{tr} \left[(M^* - AM)^T(M^* - AM)\right] ,$$

where tr denotes the trace function and a superscript T denotes transposition.

Q(A) can be expanded and rewritten as

$$Q(A) = \mathrm{tr}\left[(M^{*T} - M^T A^T)(M^* - AM)\right] = \mathrm{tr}(M^{*T} M^*) + \mathrm{tr}(M^T M) - 2\mathrm{tr}(M^T A^T M^*).$$

Since the first two terms are independent of A, Q(A) is minimized by maximizing $F(A) = \mathrm{tr}(M^T A^T M^*)$. This function, F(A), may be written as (due to the preservation of commutativity under the trace operation)

$$F(A) = \mathrm{tr}(A^T M^* M^T) .$$

It is well known that an arbitrary real-square matrix, P, can be written as a matrix product, US, where U is orthogonal and S is symmetric and positive semidefinite. Furthermore, if P is nonsingular, U is uniquely defined and S is positive definite. If P is singular, U is not unique and the problem is essentially indeterminate.

Applying the above to $P = M^* M^T$, we have $F(A) = \mathrm{tr}(A^T US)$. Since S is symmetric, there is an orthogonal matrix G such that (GSG^T) is a diagonal matrix D, whose diagonal elements d_1, ... , d_k are arranged in decreasing order. All d_j are non-negative since S is positive semidefinite. Now letting $X = (GA^T UG^T)$, we obtain

$$F(A) = \mathrm{tr}(A^T UG^T DG) = \mathrm{tr}(GA^T UG^T D) = \mathrm{tr}(XD) = \sum_{i=1}^{k} d_i x_{ii} .$$

Since F(A) is a linear function of the non-negative numbers d_1, ... , d_k, its maximum is attained when the diagonal elements of X attain their maximum values. Because X is an orthogonal

matrix, all of its elements are between -1 and +1 in value. Thus $F(A)$ is maximized when $x_{ii} = +1$ and $x_{ij} = 0$ for $i \neq j$.

Because det A is required to be +1, the det $X = \det(GA^T UG^T) = (\det G)^2(\det A)(\det U) = \det U$. If det U = -1, it is required that det X = -1 and it is not difficult to see that

$$X = \begin{pmatrix} I_{k-1} & 0 \\ 0 & -1 \end{pmatrix} = I - 2H$$

is a solution (since $d_1 \geq d_2 \geq \ldots \geq d_k$). The matrix, I, is the kth-order identity matrix and I_{k-1} is the k-1st order identity matrix. H is a kth-order square matrix with +1 as the (k, k) element and zero everywhere else. For the case where det U = +1, X = I. Now letting X_0 be the matrix that maximizes F(A) (either X = I or X = I - 2H, according to det U = +1 or -1), then

$$X_0 = G\, A_0^T\, U\, G^T, \text{ or } A_0 = U\, G^T\, X_0^T\, G$$

is a rotation matrix that minimizes the sum of squares Q(A). If $P = M^* M^T$ is nonsingular, it is the unique rotation matrix that does so.

In the solution developed by R. H. Wessner (Hughes Aircraft Co.), he points out that if det $P \neq 0$, $P = M^* M^T = US$, and

$$U = (P^T)^{-1} (P^T P)^{\frac{1}{2}}, \quad S = (P^T P)^{\frac{1}{2}} .$$

where $(P^T P)^{\frac{1}{2}}$ is the symmetric square root of $(P^T P)$ with positive eigenvalues.

Hence the solutions for A are

$$A_0 = U = (MM^{*T})^{-1} (MM^{*T} M^* M^T)^{\frac{1}{2}} \text{ for } \det P > 0, \text{ and}$$

$$A_0 = (MM^{*T})^{-1} (MM^{*T} M^{*} M^{T})^{\frac{1}{2}} (I - 2C^{T}HG) \text{ for } \det P < 0.$$

The matrix G is the modal matrix of eigenvectors for the matrix P^TP.

Appendix B
ROTATIONAL TRANSFORMATION BETWEEN LOCAL VERTICAL AND GEOCENTRIC REFERENCE SYSTEMS

The mathematical formulations used in computing the sunline vector and geomagnetic field vector are referenced to a geocentric coordinate system of axes (Z is the North Pole, X is the first point of Aries, and Y is the vector that completes the right-hand set). This means that the vectors are in geocentric coordinates rather than the desired local vertical coordinates. One way to solve this problem is to compute an orthogonal transformation that defines the orientation between the two coordinate systems.

The derivation of this matrix, denoted $\underline{\underline{C}}$, begins with the observation that the local vertical system of axes (X_ℓ, Y_ℓ, and Z_ℓ) are parallel with orbit axes whose direction cosines are a function of the standard Kepler orbit elements, Ω, ω, i, and f (defined below). These orbit axes, which will be called $\underline{X}_0$, $\underline{Y}_0$, and $\underline{Z}_0$, are identified next.

Figure B-1 shows the geometry of the satellite's orbit, as well as the location of the $\underline{X}_0$, $\underline{Y}_0$, and $\underline{Z}_0$ axes. The sequence of angular rotations from the geocentric coordinate axes to the set of orbit axes is as follows:

1. A rotation, $\underline{\underline{R}}_3(\Omega)$, about the $\underline{Z}_1$ axis is performed. The angle, Ω, is usually called the longitude of the ascending mode.

2. A rotation, $\underline{\underline{R}}_1(i)$, about the new X-axis (also called the line of nodes) is next performed. The angle, i, is the orbit inclination.

3. A rotation, $\underline{\underline{R}}_3(\omega)$, about the new Z-axis is next performed. The angle, ω, is called the argument of perigee. The new X-axis (line of apsides) of this system defines the points in orbit of the satellite's closest (perigee) and farthest (apogee) approaches to the earth's mass center.

4. The last rotation, $\underline{\underline{R}}_3(f)$, about the new Z-axis is performed. The angle, f, is called the true anomaly. The X-axis of this system intersects the orbit path at the point where the satellite's center of mass is situated.

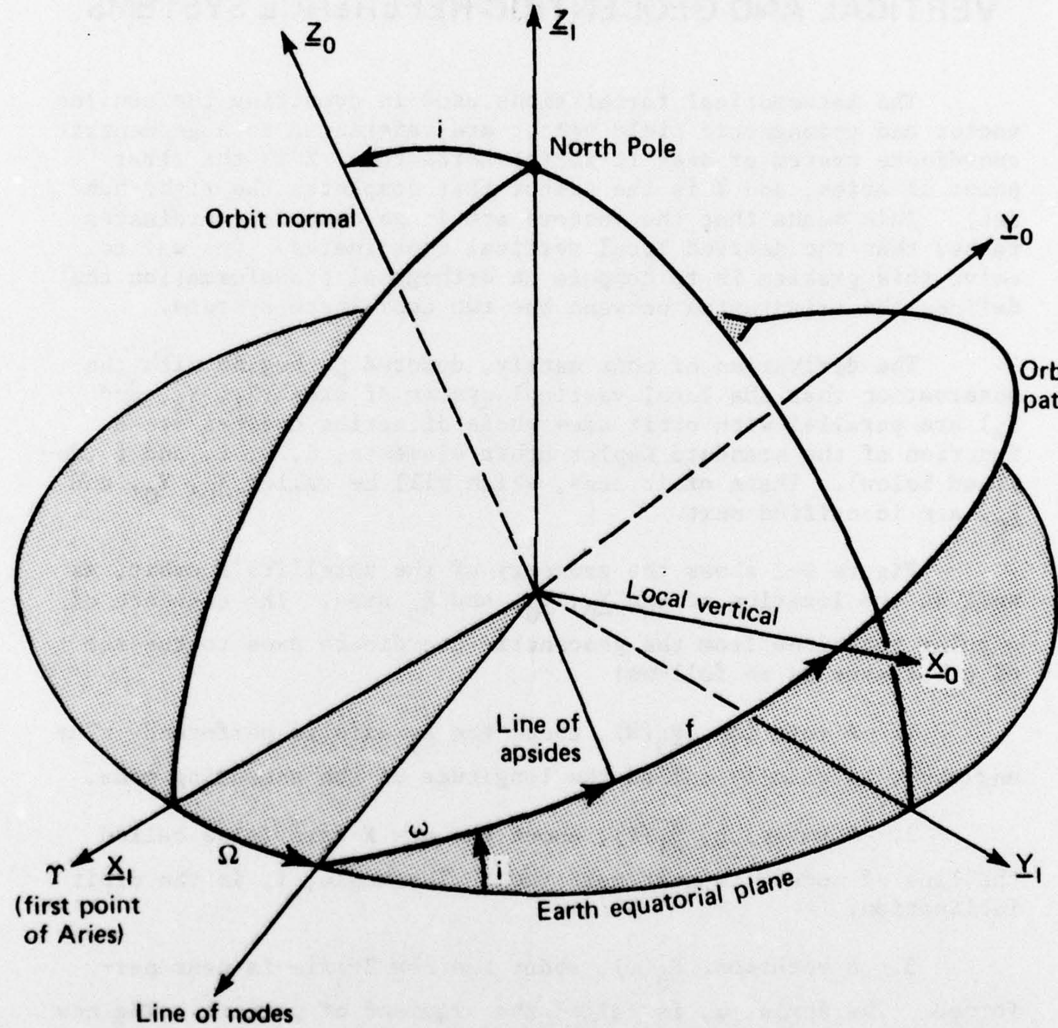

Fig. B-1 Orientation of Local Vertical System Relative to Geocentric Reference System

The product of these rotations is a direction cosine matrix, $\underline{\underline{D}}$, that can be written as

$$\underline{\underline{D}} = \underline{\underline{R}}_3(f)\underline{\underline{R}}_3(\omega)\underline{\underline{R}}_1(i)\underline{\underline{R}}_3(\Omega)$$

$$= \underline{\underline{R}}_3(F)\underline{\underline{R}}_1(i)\underline{\underline{R}}_3(\Omega) ,$$

where the last two Z-axis rotations are equivalent to a single Z-axis rotation of magnitude, $F = f + \omega$. Note that the axis, $\underline{X}_0$, is the outbound local vertical and that $\underline{Z}_0$ is the orbit normal. Identification of the local vertical axes, $\underline{X}_\ell$, $\underline{Y}_\ell$, and $\underline{Z}_\ell$ is thus equivalent to a redefinition of the $\underline{X}_0$, $\underline{Y}_0$, and $\underline{Z}_0$ axes. This can be accomplished using two 90° rotations. The first, $\underline{\underline{R}}_2(90°)$, places a Z-axis along the direction of $\underline{X}_0$. The second rotation, $\underline{\underline{R}}_3(90°)$, about the new Z-axis places the new Y-axis along the direction of $\underline{Z}_0$. Thus the total transformation matrix, $\underline{\underline{C}}$, from the geocentric reference system to the local vertical reference system is

$$\underline{\underline{C}} = \underline{\underline{R}}_3(90°)\underline{\underline{R}}_2(90°)\underline{\underline{D}}$$

or

$$\underline{\underline{C}} = \begin{pmatrix} -\Omega_c F_s - \Omega_s F_c i_c & \Omega_s i_s & \Omega_c F_c - \Omega_s F_s i_c \\ -\Omega_s F_s + \Omega_c F_c i_c & -\Omega_c i_s & \Omega_s F_c + \Omega_c F_s i_c \\ F_c i_s & i_c & F_s i_s \end{pmatrix} .$$

The determination of the orbit elements is an orbit determination problem. In the attitude estimation scheme, these are assumed known.

NOMENCLATURE

$\underline{\underline{A}}$ = Orthogonal transformation matrix from a local vertical reference system to the reference axes fixed in the satellite

$\underline{\underline{C}}$ = Orthogonal transformation matrix from the geocentric coordinate system to the local vertical reference system

Ω = Orbit longitude of the ascending node

ω = Orbit argument of perigee

i = Orbit inclination

f = Orbit true anomaly

Special Notation

$$\underline{\underline{R}}_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha_c & \alpha_s \\ 0 & -\alpha_s & \alpha_c \end{bmatrix}$$ = matrix representation of a positive rotation of α radians about a 1- or X-axis

$$\underline{\underline{R}}_2(\alpha) = \begin{bmatrix} \alpha_c & 0 & -\alpha_s \\ 0 & 1 & 0 \\ \alpha_s & 0 & \alpha_c \end{bmatrix}$$ = matrix representation of a positive rotation of α radians about a 2- or Y-axis

$$\underline{\underline{R}}_3(\alpha) = \begin{bmatrix} \alpha_c & \alpha_s & 0 \\ -\alpha_s & \alpha_c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ = matrix representation of a positive rotation of α radians about a 3- or Z-axis

α_s = Sine of α

α_c = Cosine of α

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$\underline{u}_v$ = Column vector, $\underline{u}$, in the satellite coordinate system

$\underline{u}_I$ = Column vector, $\underline{u}$, in the geocentric coordinate system

$\underline{u}_\ell$ = Column vector, $\underline{u}$, in the local vertical coordinate system

$\underline{u}_0$ = Column vector, $\underline{u}$, in the orbit coordinate system

$\underline{\underline{U}}^T$ = Transpose of the matrix, $\underline{\underline{U}}$ (applies to column vectors also)

$\underline{\underline{U}}^{-1}$ = Inverse of the square matrix, $\underline{\underline{U}}$

$\mathrm{tr}\ \underline{\underline{U}} = \sum_{j=1}^{k} u_{jj}$ = trace function of the kth order matrix, $\underline{\underline{U}}$

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The work reported in TG 1313 was done under Navy Contract N00017-72-C-4401. This work is related to Task S1T1, which is supported by Strategic Systems Project Office (SP-24).

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